ARITHMETICS OF RATIONAL GENERALIZED QUATERNION ALGEBRAS

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1. **Introduction.** A rational generalized quaternion algebra is a linear associative algebra of order four and rank two, having a principal modulus I_0, ranging over the field of rational numbers. A basis (I_0, I_1, I_2, I_1I_2) may be found[1] such that

$$I_1^2 = \alpha, \quad I_2^2 = \beta, \quad I_1I_2 = -I_2I_1, \qquad \alpha \text{ and } \beta \text{ rational integers.} \tag{1}$$

An algebra in such form will be denoted by $Q(\alpha, \beta)$, a number in it by

$$Q = a_0I_0 + a_1I_1 + a_2I_2 + a_{12}I_1I_2, \qquad a\text{'s rational,} \tag{2}$$

and its norm by

$$N(Q) = a_0^2 - \alpha a_1^2 - \beta a_2^2 + \alpha\beta a_{12}^2. \tag{3}$$

Q satisfies its rank equation

$$X^2 - 2a_0X + N(Q) = 0. \tag{4}$$

a_0 is called the real part of Q.

An arithmetic S of $Q(\alpha, \beta)$ is a set of numbers having the following properties:

C_a: S is closed with respect to algebraic addition.

C_m: S is closed with respect to multiplication.

R: For every number of S, (4) has integral coefficients.

U: S contains I_0, I_1 and I_2 (and hence I_1I_2 by C_m).

M: S is maximal; that is, S is contained in no larger set having Properties C_a, C_m, R and U.

It is the purpose of this paper to determine a set of bases for the arithmetics of those algebras for which α and β contain no squared prime factors.[2] In any case, it has been proved[3] that for a given arith-

[1] Dickson, L. E., *Algebren und ihre Zahlentheorie*, Zurich, 1927, pp. 43–44. The definition of an arithmetic is made there also.

[2] If α and β contain squared prime factors, the number of arithmetics varies with the form of those factors. See M. Eichler, *Untersuchungen in der Zahlentheorie der rationalen Quaternionalgebren*, Journal für die reine und angewandte Mathematik, vol. 174 (1936), p. 149, Theorem 12.

[3] Latimer, C. G., *The classes of integral sets in a quaternion algebra*, Duke Mathematical Journal, vol. 3 (1937), pp. 246–247, §7. On pages 237–238, §2, of this reference is stated a theorem giving necessary and sufficient conditions that a basis of $Q(\alpha, \beta)$ be a basis of an *arithmetic* of $Q(\alpha', \beta')$.

metic S in the algebra, a basis $(I_0', I_1', I_2', I_1'I_2')$ can be found such that $I_1'^2=\alpha', I_2'^2=\beta', I_1'I_2'=-I_2'I_1', \alpha'$ and β' containing no squared prime factors, and such that the given arithmetic is an arithmetic of $Q(\alpha', \beta')$. The problem of determining bases for the arithmetics of such algebras has been solved by Latimer[4] when $\alpha\equiv\beta\equiv 1 \pmod 2$, and by Darkow[5] when $\alpha\equiv\beta\equiv 0 \pmod 2$. Darkow's results can be extended to apply to the remaining cases (when $\alpha+\beta\equiv 1 \pmod 2$).[6] All possible cases are treated in this paper, with greatly simplified results.[7]

Albert[8] has shown that every rational generalized quaternion *division* algebra $Q(\alpha, \beta)$ may be transformed into $Q(\tau, \sigma)$, in which τ and σ have special properties. Concerning the treatments by Latimer and Darkow (similar to that used in this paper) Albert writes:[9]

> The · · · division into special cases is certainly not desirable. Nor is it necessary. For it is obvious that at least an attempt should be made to show that transformations carrying all cases into a canonical form are possible, and it is this canonical form which should be studied. · · · In particular, it is evident that the integral sets of S should contain the integers of the realm $R(i)$.

Concerning the above quotation, a few comments seem pertinent. Albert's results are readily obtainable from the results of this paper or those of Latimer and Darkow. His canonical form is not unique, and the number (two) of arithmetics obtained by it is, in general, much less than the number of arithmetics as defined by this paper. There being no essential difference between the basal elements I_1 and I_2, or, for that matter, I_1I_2, there seems to be little reason for requiring that the integers of the realm $R(I_1)$ be contained in an integral set (arithmetic) in preference to those of realms $R(I_2)$ or $R(I_1I_2)$. Albert's results apply only to division algebras, while those of Latimer and Darkow and the writer apply to all algebras. The norm of a number of $Q(\alpha, \beta)$, given in (3), being of a form of great interest in number theory, it would seem particularly useful to have

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[4] Latimer, C. G., *Arithmetics of generalized quaternion algebras*, American Journal of Mathematics, vol. 48 (1926), pp. 57–66.

[5] Darkow, M. D., *Determination of a basis for the integral elements of certain generalized quaternion algebras*, Annals of Mathematics, (2), vol. 28 (1926), pp. 263–270.

[6] Latimer, C. G., *On the class number of a quaternion algebra with a negative fundamental number*, Transactions of this Society, vol. 40 (1936), p. 320.

[7] The writer is greatly indebted to Dr. Claiborne G. Latimer for valuable suggestions enabling much greater simplification than he had previously obtained.

[8] Albert, A. A., *Integral domains of rational generalized quaternion algebras*, this Bulletin, vol. 40 (1934), pp. 164–176.

[9] Albert, A. A., loc. cit., p. 165. The i used there is the same as the I used in this paper.

available a method of obtaining the arithmetics of an algebra expressed in terms of parameters α and β which yield that exact norm. Clearly Albert's canonical form would not suffice, so that methods of treatment as given here are conceivably not only useful but necessary.

As mentioned above, the method of treatment in this paper is similar to that used by Latimer and Darkow, the work being more direct in some places.[10]

2. **Notations, transformations and definitions used.** The letters r, s and t will be used exclusively to denote permutations of 1, 2, and 3. Let $(g-1)(g+1)=0$ such that

$$g = \begin{cases} 1 \text{ if the number of inversions of } (rst) \text{ is even,} \\ -1 \text{ otherwise.} \end{cases} \tag{5}$$

Let the g.c.d. of α and β be

$$(\alpha, \beta) = \gamma_3 \text{ (positive or negative)} \tag{6}$$

and $\alpha = -\gamma_2\gamma_3$, $\beta = \gamma_3\gamma_1$. Let

$$I_1 I_2 = \gamma_3 I_3, \tag{7}$$

so that from (1), $I_r^2 = -\gamma_s\gamma_t$, and $I_r I_s = g\gamma_t I_t$. From the results of Latimer and Darkow, it can easily be shown that I_3 is in each arithmetic, as are I_1 and I_2. Hence from (6) and (7), I_1, I_2, and I_3 are formaliy identical, as are the γ's, the latter being formally identical to their negatives. $Q(\alpha, \beta)$ will hereafter be written in the symmetric notation

$$Q(\gamma_1, \gamma_2, \gamma_3) \equiv Q(\alpha, \beta). \tag{8}$$

Making the unitary transformation

$$\begin{gathered} i_0 = gI_0, \qquad i_1 = gI_r, \qquad i_2 = gI_s, \qquad i_3 = gI_t, \\ \gamma_r = \alpha_1, \qquad \gamma_s = \alpha_2, \qquad \gamma_t = \alpha_3, \end{gathered} \tag{9}$$

one has $i_r^2 = -\alpha_s\alpha_t$ and $i_r i_s = g\alpha_t i_t$, which is abstractly identical with (9). $Q(\gamma_1, \gamma_2, \gamma_3)$ now becomes $Q(\alpha_1, \alpha_2, \alpha_3)$. After proper change of sign of the γ's, one finds that γ_t is odd or even, $\gamma_s \equiv 1 \pmod 4$, γ_r is odd, and $\gamma_r \equiv 1 \pmod 4$ if γ_t is odd. Choosing

$$(\theta - 1)(\theta - 2) = 0, \tag{10}$$

$Q(\alpha_1, \alpha_2, \alpha_3)$ is such that

[10] Latimer and Darkow obtain their results for $Q(\alpha, -\beta)$. Each mentions a "tentative" process for obtaining bases. See Latimer, loc. cit., footnote 3, pp. 61 and 65; Darkow, loc. cit., p. 263 and p. 267 ff.

(11) $\alpha_3 \equiv \theta \pmod{2\theta}$, $\alpha_2 \equiv 1 \pmod 4$, $\alpha_1 \equiv \alpha_2 \pmod{2(3-\theta)}$.

In what follows, it will be assumed that (9) has been made so that (11) holds. All other symbols introduced hereafter will represent integers, unless they are obviously not integers. The parameters θ, m, k, n, A_r, B_r, ξ and H_j, used frequently hereafter, and most easily determined in the order given, are defined by the following, each being unique save n and H_j:

θ: $\alpha_3 \equiv \theta \pmod{2\theta}$; $(\theta-1)(\theta-2)=0$.

m: $(\alpha_1+\alpha_2+m\alpha_3) \equiv 2-\theta \pmod{4\theta}$; $(m-1)(m+1)[(\theta-1)m(m+2)+2-\theta]=0$.

k: $2(m^2+m+1)k=(m+1)(m+2)$.[11]

n: $kn^2-n=0$.

A_r, B_r: $\alpha_r=A_rB_r$, and

$$\left(\frac{-\alpha_s\alpha_t}{A_r}\right)=1, \qquad \left(\frac{-\alpha_s\alpha_t}{B_r}\right)=-1,$$

B_r positive and a minimum.[12]

ξ: ξ is the number of prime factors (all distinct) of $A_1A_2A_3$.

H_j: $(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)H_j^2 \equiv -1 \pmod{A_1A_2A_3}$, $(j=1, 2, \cdots, \xi)$.[13] If H_j is even, then $H_j'=H_j \pm A_1A_2A_3$ is odd. Let $f_7=(1-2m)$ and $f_8=2\theta k-2k+2$. Then if $A_2B_1H_j' \not\equiv f_7 \pmod{f_8}$, it can be shown that $H_j''=H_j' \pm A_1A_2A_3$ is such that $A_2B_1H_j'' \equiv f_7 \pmod{f_8}$. Since H_j, H_j', and H_j'' are all in the same residue class, modulo $A_1A_2A_3$, it follows (and will be so assumed hereafter) that H_j may always be chosen such that (f_7 is odd and f_8 is even, so H_j will be odd)

$$A_2B_1H_j \equiv f_7 \pmod{f_8}. \tag{12}$$

The following symbols will also be used:

$$\begin{aligned} &f_1=(2-\theta)kn, \quad f_2=(\theta n-2n+2), \quad f_3=k(m+\theta n-2n), \\ &f_4=(2-\theta)(m-2n-k+1)+n, \quad f_5=(2n-\theta n-k+2), \\ &f_6=(\theta-1)(m-2k+2), \quad f_7=(1-2m), \quad f_8=2\theta k-2k+2, \end{aligned} \tag{13}$$

[11] If $e=1-k$, then k may also be defined by $(\alpha_1+\alpha_2+\alpha_3)(\alpha_1+\alpha_2)(\alpha_2+\alpha_3)(\alpha_3+\alpha_1) \equiv 8e \pmod{16}$. See Latimer, C. G., *On the fundamental number of a rational generalized quaternion algebra*, Duke Mathematical Journal, vol. 1 (1935), p. 435, theorem and second footnote.

[12] In these conditions

$$\left(\frac{A}{P}\right)$$

is the Legendre symbol for quadratic residues.

[13] This congruence has exactly ξ distinct solutions. See Dickson, L. E., *Introduction to the Theory of Numbers*, University of Chicago Press, 1929, p. 12, Theorem 16.

$$(14)\quad M_j = \frac{\alpha_1 H_j i_1 + i_2}{2A_3}, \qquad N_j = \tfrac{1}{2} i_0 + \frac{-\alpha_1 H_j i_1 + \alpha_2 H_j i_2 + i_3}{2A_1 A_2}.$$

3. **The fundamental form of a number Q in an arithmetic.** From the relations given by (2), (3), (7) and (9), a number in $Q(\alpha_1, \alpha_2, \alpha_3)$ is such that

$$(15)\quad \begin{aligned} Q &= b_0 i_0 + b_1 i_1 + b_2 i_2 + b_3 i_3, \\ N(Q) &= b_0^2 + \alpha_2\alpha_3 b_1^2 + \alpha_3\alpha_1 b_2^2 + \alpha_1\alpha_2 b_3^2 \end{aligned}$$

(the b's rational). By Property C_m, if Q is in S, then so are Qi_0, Qi_1, Qi_2 and Qi_3, whose real parts are b_0, $-\alpha_2\alpha_3 b_1$, $-\alpha_3\alpha_1 b_2$ and $-\alpha_1\alpha_2 b_3$, respectively, and by Property R and (4), one may write $2b_0 = X_0$, $2b_r\alpha_s\alpha_t = X_r$, so that (15) becomes

$$(16)\quad \begin{aligned} Q &= \frac{1}{2}\left[X_0 i_0 + \frac{X_1 i_1}{\alpha_2\alpha_3} + \frac{X_2 i_2}{\alpha_3\alpha_1} + \frac{X_3 i_3}{\alpha_1\alpha_2}\right], \\ N(Q) &= \frac{1}{4}\left[X_0^2 + \frac{X_1^2}{\alpha_2\alpha_3} + \frac{X_2^2}{\alpha_3\alpha_1} + \frac{X_3^2}{\alpha_1\alpha_2}\right] \end{aligned}$$

and, by Property R,

$$(17)\quad \alpha_1\alpha_2\alpha_3 X_0^2 + \alpha_1 X_1^2 + \alpha_2 X_2^2 + \alpha_3 X_3^2 \equiv 0 \pmod{4\alpha_1\alpha_2\alpha_3}.$$

Using (11), (17) becomes equivalent to

$$(18)\quad \begin{aligned} &\alpha_1\alpha_2\alpha_3 X_0^2 + \alpha_1 X_1^2 + \alpha_2 X_2^2 + \alpha_3 X_3^2 \equiv 0 \pmod{4\theta} \\ &\qquad \text{if also } \alpha_r X_r^2 + \alpha_s X_s^2 \equiv 0 \pmod{\alpha_t}. \end{aligned}$$

Let δ_t be any prime factor of α_t. If δ_t divides X_r, it also divides X_s. But if $(X_r, \delta_t) = 1$, then there exists an integer p_r such that $p_r X_r \equiv 1 \pmod{\delta_t}$. Hence from (18), $p_r^2\alpha_r X_r^2 + p_r^2\alpha_s X_s^2 \equiv 0 \pmod{\delta_t}$, $\alpha_r + p_r^2\alpha_s X_s^2 \equiv 0 \pmod{\delta_t}$, or $(p_r\alpha_s X_s)^2 \equiv -\alpha_r\alpha_s \pmod{\delta_t}$. Hence

$$\left(\frac{-\alpha_r\alpha_s}{\delta_t}\right) = 1,$$

and from (13), X_r may be prime to each factor of A_t, but must be divisible by each factor of B_t, and hence by B_t itself. Therefore B_t divides X_r and X_s, and one may write

$$(19)\quad X_0 = x_0, \qquad X_r = B_s B_t x_r,$$

and (16) and (18) become, respectively,

$$(20)\qquad Q_z = \frac{1}{2}\left[x_0 i_0 + \frac{x_1 i_1}{A_2 A_3} + \frac{x_2 i_2}{A_3 A_1} + \frac{x_3 i_3}{A_1 A_2}\right],$$

$$N(Q) = \frac{1}{4}\left[x_0^2 + \frac{B_2 B_3 x_1^2}{A_2 A_3} + \frac{B_3 B_1 x_2^2}{A_3 A_1} + \frac{B_1 B_2 x_3^2}{A_1 A_2}\right],$$

and

$$(21)\qquad \alpha_1\alpha_2\alpha_3 x_0^2 + \alpha_1 x_1^2 + \alpha_2 x_2^2 + \alpha_3 x_3^2 \equiv 0 \pmod{4\theta},$$

$$(A_s B_r x_s)^2 \equiv -\alpha_r\alpha_s x_r^2 \pmod{A_t}.$$

Hereafter, the numbers x_0, x_1, x_2, x_3 will be referred to as the *coordinates of a number Q_z in S*, and (20) will be called the *fundamental form of Q in S*. Using the definition of H_j to obtain $\alpha_r\alpha_s H_j^2 \equiv -1 \pmod{A_t}$, and multiplying both members of (21b) by H_j^2, one obtains $(A_s B_r H_j x_s)^2 \equiv -\alpha_r\alpha_s H_j^2 x_r^2 \equiv x_r^2 \pmod{A_t}$, or, reducing to linear congruences, $x_r \equiv A_s B_r(\pm H_j)x_s \pmod{A_t}$. Now if H_j is a solution of its congruence, then $-H_j$ is another, distinct from H_j, and one may finally write

$$(22)\qquad x_r \equiv gA_s B_r H_j x_s \pmod{A_t},$$

where g is defined as in (5). Hence from (20), (21) and (22) one concludes:

THEOREM I. *If a number Q is in an arithmetic, it is necessary that it have the form* (20), *whose coordinates satisfy* (21a) *and* (22).

4. **Analysis of the conditions imposed on the coordinates of a number Q of fundamental form lying in an arithmetic.** It can be shown that two numbers of fundamental form have Property C_a if and only if their coordinates satisfy (22) *for the same value of j*. Hereafter H_j will be replaced by H whenever the value of j is fixed in an argument. Let Q_z and Q_y be in an arithmetic, and hence satisfy (22) for the same value of j. Then by Property C_m, $Q_z Q_y$ must also lie in S. A study of the coordinates of this new number reveals that in addition to (21a) and (22), the following conditions must hold:[14]

$$(23)\qquad \begin{aligned} \alpha_1\alpha_2\alpha_3 x_0 y_0 + \alpha_1 x_1 y_1 + \alpha_2 x_2 y_2 + \alpha_3 x_3 y_3 &\equiv 0 \pmod{2\theta},\\ x_0 y_1 + y_0 x_1 + y_2 x_3 + x_2 y_3 &\equiv 0 \pmod 2,\\ x_0 y_2 + y_0 x_2 + y_3 x_1 + x_3 y_1 &\equiv 0 \pmod 2,\\ \theta(x_0 y_3 + y_0 x_3) + y_1 x_2 - x_1 y_2 &\equiv 0 \pmod{2\theta}. \end{aligned}$$

[14] Here begins the direct method, as opposed to the "tentative" process of Latimer and Darkow. See footnote 10.

Exhaustive analysis of (21a), (22) and (23) further reveals that the conditions that must hold in order for Q_x and Q_y *to have property M* are as follows:

$$
\begin{aligned}
x_0 &= [(2-\theta)n + (\theta-1)km]x_1 \\
&\quad + (2-\theta)(1-n)x_2 + (\theta-1)x_3 \pmod 2, \\
x_1 &\equiv 0 \pmod{\theta + k - \theta k}, \\
x_1 &\equiv (1-2n)x_2 + 2mx_0 \pmod{\theta k + 2\theta - 2k}, \text{ for fixed } n, \\
x_3 &\equiv (2-\theta)[(1-n)x_1 + nx_2] \pmod{3-\theta}
\end{aligned} \tag{24}
$$

$$
x_1 \equiv B_1H(A_2x_2 - A_3x_3) \pmod{A_2A_3}, \qquad x_2 \equiv A_3B_2Hx_3 \pmod{A_1}, \tag{25}
$$

which is equivalent to (22). Letting P_0, P_1, P_2, and P_3 be arbitrary integers, the f's integers as defined in (13), and M_j and N_j as in (14), then (24) and (25) can be reduced to

$$
\begin{aligned}
x_0 &= 2P_0 + f_1P_1 + f_3P_2 + P_3, \\
x_1 &= f_2A_2A_3P_1 + (f_4A_2A_3 + f_5A_2\alpha_1H)P_2 + (f_6A_2A_3 - A_3B_1H)P_3, \\
x_2 &= f_5A_1P_2 + A_3B_2HP_3, \\
x_3 &= P_3, \qquad \text{for fixed } n.
\end{aligned} \tag{26}
$$

Substituting these in (20a) one has

$$
\begin{aligned}
Q_{xjn} &= P_0i_0 + \tfrac{1}{2}(f_1i_0 + f_2i_1)P_1 + [\tfrac{1}{2}(f_3i_0 + f_4i_1) + f_5M_j]P_2 \\
&\quad + [\tfrac{1}{2}f_6i_1 + N_j]P_3, \qquad j = 1, 2, \cdots, \xi.
\end{aligned} \tag{27}
$$

Now let

$$
\begin{aligned}
U_{0jn} &= i_0, & U_{1jn} &= \tfrac{1}{2}(f_1i_0 + f_2i_1), \\
U_{2jn} &= \tfrac{1}{2}(f_3i_0 + f_4i_1) + f_5M_j, & U_{3jn} &= \tfrac{1}{2}f_6i_1 + N_j.
\end{aligned} \tag{28}
$$

(27) then becomes

$$
Q_{xjn} = P_0U_0 + P_1U_1 + P_2U_2 + P_3U_3. \tag{29}
$$

All such numbers, for a fixed j and n, form an integral set of numbers with (28) as a basis. Since (28) is equivalent to (26) and (20) combined, and since (26) and (20) establish necessary and sufficient conditions that Properties C_a, C_m, R and M hold, and it is easily shown that i_1, i_2 and i_3 are of the form (29), so that Property U holds, it follows that (28) gives the bases of the algebra—one for each value of j and n. But n has $k+1$ values, and j has ξ values. By use of the inverse of the unitary transformation (9), and of (7), it is clear that the arith-

metics of $Q(\alpha_1, \alpha_2, \alpha_3)$ are also the arithmetics of $Q(\gamma_1, \gamma_2, \gamma_3)$ and of $Q(\alpha, \beta)$. Hence

THEOREM II. *The numbers of* (28) *constitute the bases for the arithmetics of* $Q(\alpha_1, \alpha_2, \alpha_3)$, *of* $Q(\gamma_1, \gamma_2, \gamma_3)$, *and of* $Q(\alpha, \beta)$. *There are* $2^k\xi$ *such arithmetics.*

The number of arithmetics agrees with that determined by Latimer and Darkow, and the bases were checked by applying the theorem mentioned in footnote 3. In this theorem the fundamental number d of the algebra is used. By a known result[11]

$$d = \pm 2^{1-k} B_1 B_2 B_3,$$

the sign being positive if $\alpha_1, \alpha_2, \alpha_3$ are all of the same sign, and negative otherwise; that is, according as $N(Q)$ (see (3)) is a definite or indefinite form. It is also known that $Q(\alpha, \beta)$ is a division algebra if and only if $d \neq -1$.[15]

5. **The method of writing down the arithmetics of $Q(\alpha, \beta)$, or of $Q(\gamma_1, \gamma_2, \gamma_3)$, α and β containing no squared prime factors.** By use of (7) and (9), and a few new symbols, the arithmetics may readily be written down in terms of the original basis, (I_0, I_1, I_2, I_1I_2). Let the following definitions and order of procedure be made:

(a) $(\alpha, \beta) = \gamma_3$ (positive or negative), so that $\alpha = -\gamma_2\gamma_3$, $\beta = -\gamma_3\gamma_1$.

(b) $\gamma_r = \Gamma_r\Delta_r$, and

$$\left(\frac{-\gamma_s\gamma_t}{\Gamma_r}\right) = 1, \qquad \left(\frac{-\gamma_s\gamma_t}{\Delta_r}\right) = -1,$$

Δ_r a positive minimum and (rst) a permutation of (123).

(c) Arrange $\gamma_1, \gamma_2, \gamma_3$ (changing signs of all simultaneously if necessary) and determine θ so that $\gamma_t \equiv \theta \pmod{2\theta}$, $\gamma_s \equiv 1 \pmod 4$, $\gamma_r \equiv \gamma_s \pmod{2(3-\theta)}$.

(d) Then determine m so that $\gamma_r + \gamma_s + m\gamma_t \equiv 2-\theta \pmod{4\theta}$, $(m-1)(m+1)[(\theta-1)m(m+2)+2-\theta] = 0$.

(e) Determine k and n such that $2(m^2+m+1)k = (m+1)(m+2)$, and $kn^2 - n = 0$.

(f) Determine $f_1, f_2, \cdots, f_8$ according to Table I.

(g) Determine the ξ solutions H_j $(j = 1, 2, \cdots, \xi)$ of $(\gamma_1\gamma_2 + \gamma_2\gamma_3 + \gamma_3\gamma_1)H_j^2 \equiv -1 \pmod{\Gamma_1\Gamma_2\Gamma_3}$, ξ being the number of prime factors of $\Gamma_1\Gamma_2\Gamma_3$. If any H_j is even, replace it by $H_j \pm \Gamma_1\Gamma_2\Gamma_3$ to make it odd.

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[15] Latimer, C. G., loc. cit., footnote 11. See §1, next to last paragraph, and the corollary to the theorem on page 435.

If then any H_j be such that $\Gamma_s\Delta_r H_j \not\equiv f_7 \pmod{f_8}$, replace it by $H_j \pm \Gamma_1\Gamma_2\Gamma_3$ and find that $\Gamma_s\Delta_r H_j \equiv f_7 \pmod{f_8}$.

(h) Then determine V_{0jn}, V_{1jn}, V_{2jn}, V_{3jn}, as the desired bases, for each value of j and n, where

$$V_{0jn} = I_0, \qquad V_{1jn} = \frac{f_1 I_0 + f_2 I_r}{2},$$

$$V_{2jn} = \frac{f_3 I_0 + f_4 I_r}{2} + f_5 \frac{\gamma_r H_j I_r + I_s}{2\Gamma_t}, \tag{30}$$

$$V_{3jn} = \frac{I_0 + f_6 I_r}{2} + \frac{-\gamma_r H_j I_r + \gamma_s H_j I_s + I_t}{2\Gamma_r\Gamma_s}.$$

(i) Finally, replace $\alpha_3 I_3$ by $I_1 I_2$, giving the final forms.

Hull[16] has solved the problem of determining a set of bases for all

TABLE I

θ	1	1	1	2	2	2	2	2	2
m	−1	1	1	1	1	0	0	−1	−2
k	0	1	1	1	1	1	1	0	0
n	0	0	1	0	1	0	1	0	0
f_1	0	0	1	0	0	0	0	0	0
f_2	2	2	1	2	2	2	2	2	2
f_3	0	1	0	1	1	0	0	0	0
f_4	0	1	0	0	1	0	1	0	0
f_5	2	1	2	1	1	1	1	2	2
f_6	0	0	0	1	1	0	0	1	0
f_7	3	−1	−1	−1	−1	1	1	3	5
f_8	2	2	2	4	4	4	4	2	2

the arithmetics of *division* algebras, by showing that every arithmetic may be obtained from some one of an infinite number of canonical generations of the algebra (merely a non-unitary transformation of units carrying α and β into α' and β' having special properties). Latimer has obtained similar results without the restriction that the algebras be division algebras.[3] However, the method of determining which canonical generations will give the arithmetics for a given un-

[16] Hull, Ralph, *The maximal orders of generalized quaternion division algebras*, Transactions of this Society, vol. 40 (1936), pp. 1–11.

altered α and β is not stated in either paper. Hull showed that the canonical forms determined by Albert[8] are special cases of his canonical generations.

One problem remains to be solved, namely a method of writing down the arithmetics of $Q(\alpha, \beta)$ when α and β contain squared prime factors.

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A NOTE ON MAXIMUM MODULUS AND THE ZEROS OF AN INTEGRAL FUNCTION

S. M. SHAH

Let $f(z)$ be an integral function of finite order $\rho \geqq 0$, let $M(r, f) = M(r) = \max_{|z|=r} |f(z)|$, and let $n(r, f) = n(r)$ be the number of zeros of $f(z)$ in $|z| \leqq r$ and on its circumference. I have discussed elsewhere[1] the behaviour of $g(r) = \log M(r)/n(r)$, as $r \to \infty$, and have proved that *for every canonical product function* $f(z)$

$$\liminf_{r=\infty} \frac{\log M(r)}{n(r)\phi(r)} = 0, \tag{1}$$

where $\phi(r)$ *is any positive L function*[2] *such that*

$$\int^{\infty} \frac{dx}{x\phi(x)} < A = \text{const.} \tag{2}$$

The question arises how large and how small $g(r)$ and $G(r) = T(r, f)/n(r)$ can be, where $T(r, f) = T(r)$ is the Nevanlinna characteristic function for $f(z)$. I prove in this note the following result.

THEOREM. *Given* $\rho \geqq 0$ *and* $\psi(x)$ *any positive function such that*

$$\limsup_{x=\infty} \frac{\log \psi(x)}{\log x} \leqq \rho.$$

There exists an integral function $F(z)$ *of order* ρ *for which*

$$\limsup_{r=\infty} \frac{T(r, F)}{\psi(r)n(r, F)} = \infty \tag{3}$$

and an integral function $f(z)$ *of order* ρ *for which*

$$\liminf_{r=\infty} \frac{\psi(r)T(r, f)}{n(r, f)} = 0. \tag{4}$$

PROOF. We shall first construct an integral function $f(z)$ of order ρ for which

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[1] (i) *A theorem on integral functions of integral order*, Journal of the London Mathematical Society, vol. 15 (1940), pp. 23–31. (ii) *On integral functions of integral or zero order*, to be published. (iii) *On integral functions of perfectly regular growth*, Journal of the London Mathematical Society, vol. 14 (1939), pp. 293–302.

[2] For definition see G. H. Hardy, *Orders of Infinity*, 1924, p. 17.

$$\liminf_{r=\infty} \frac{\log M(r, f)\psi(r)}{n(r, f)} = 0 \tag{4.1}$$

from which (4) will follow. Let

$$\lambda_n = n^{n^n}, \qquad \psi(x) = e^{(\rho+\eta')\log x}$$

where we may suppose without loss of generality that $\eta_x' > 0$. We know that $\eta_x' \to 0$ as $x \to \infty$. Let η_r be the upper bound of η_t' for $t \geqq r$. Then $\eta_r \to 0$ monotonically as $r \to \infty$, and $\psi(r) \leqq e^{(\rho+\eta_r)\log r}$. Let $\epsilon_1 = \epsilon_2 = 1$, and

$$\epsilon_n = 2\eta_n + \frac{(4+\rho)\log \lambda_{n-1}}{\log \lambda_n}, \qquad n = 3, 4, 5, \cdots;$$

$$k = [\rho] + 1;$$

$$\mu_n = [\lambda_n^{\rho+\epsilon_n}], \quad \zeta_n = [\lambda_n^{\rho/k+\epsilon_n}], \quad n = 1, 2, 3, \cdots;$$

$$f(z) = \prod_1^\infty \left\{1 + \left(\frac{z}{\lambda_n}\right)^{k\mu_n}\right\}; \qquad F(z) = \prod_1^\infty \left\{1 + \frac{z^k}{\lambda_n}\right\}^{\zeta_n}.$$

It is easily seen that $f(z)$ and $F(z)$ are canonical products. Further, $n(\lambda_n, f) = k(\mu_1 + \cdots + \mu_n) \sim k\mu_n$ and hence

$$\limsup_{r=\infty} \frac{\log n(r, f)}{\log r} = \limsup_{n=\infty} \frac{\log k + \log \mu_n}{\log \lambda_n} = \rho.$$

Hence $f(z)$ is an integral function of order ρ. Let now $x = \lambda_m$. Then

$$\log f(x) = \sum_{n=1}^{m-1} \log \left\{1 + \left(\frac{\lambda_m}{\lambda_n}\right)^{k\mu_n}\right\} + \log 2 + \sum_{m+1}^{\infty} \log \left\{1 + \left(\frac{\lambda_m}{\lambda_n}\right)^{k\mu_n}\right\}$$

$$= \Sigma_1 + \log 2 + \Sigma_3.$$

We have

$$\Sigma_3 \leqq \sum_{m+1}^{\infty} \left(\frac{\lambda_m}{\lambda_n}\right)^{k\mu_n} \leqq \sum_{m+1}^{\infty} \left(\frac{1}{2}\right)^{k\mu_n} = O(1),$$

$$\Sigma_1 < m \log \left\{2(\lambda_m)^{k\mu_{m-1}}\right\}.$$

Hence for $m \geqq m_0$,

$$\log f(x) < 2m\left\{\log 2 + k\mu_{m-1} \log \lambda_m\right\}$$

$$\leqq \exp\left\{\log \mu_{m-1} + 2 \log \log \lambda_m\right\},$$

$$\frac{\psi(x)\log f(x)}{n(x,f)} \leqq \exp\{(\rho+\eta_{\lambda_m})\log\lambda_m + \log\mu_{m-1} + 2\log\log\lambda_m - \log\mu_m\}$$
$$\leqq \exp\{(\rho+\eta_m)\log\lambda_m + \log\mu_{m-1} + 2\log\log\lambda_m$$
$$- (\rho + 2\eta_m + (4+\rho)\log\lambda_{m-1}/\log\lambda_m)\log\lambda_m + O(1)\},$$

and the last expression tends to zero with $1/m$. Hence

$$\liminf_{r=\infty} \frac{\psi(r)\log M(r,f)}{n(r,f)} = 0$$

which proves (4.1). Further

$$n(\lambda_n^{1/k}, F) = k(\zeta_1 + \cdots + \zeta_n) \sim k\zeta_n.$$

Hence $F(z)$ is an integral function of order ρ. Let now $r = \frac{1}{2}\lambda_n^{1/k}$. Then

$$n(r,F) = k(\zeta_1 + \cdots + \zeta_{n-1}) \leqq k\zeta_{n-1}\left(1 + \frac{2\zeta_{n-2}}{\zeta_{n-1}}\right)$$
$$< \exp\left\{\log k + \log\zeta_{n-1} + \frac{2\zeta_{n-2}}{\zeta_{n-1}}\right\},$$
$$\log M(\tfrac{2}{3}r, F) > \zeta_n \log\left(1 + \frac{1}{3^k}\right)$$
$$= A\exp\left\{\left(\frac{\rho}{k} + \epsilon_n\right)\log\lambda_n + O(\lambda_n^{-\rho/k})\right\},$$

for $n \geqq n_0$. Hence[3]

$$\frac{\log M(\frac{2}{3}r, F)}{n(r,F)\psi(r)} > A\exp\left\{\left(\frac{\rho}{k} + \epsilon_n\right)\log\lambda_n - \log\zeta_{n-1}\right.$$
$$\left. - (\rho+\eta_n)\left(\frac{1}{k}\log\lambda_n - \log 2\right) + O(1)\right\}$$
$$= A\exp\left\{\epsilon_n\log\lambda_n - \log\zeta_{n-1} - \frac{\eta_n}{k}\log\lambda_n + O(1)\right\}$$
$$= A\exp\left\{2\eta_n\log\lambda_n + (4+\rho)\log\lambda_{n-1}\right.$$
$$\left. - \left(\frac{\rho}{k} + \epsilon_{n-1}\right)\log\lambda_{n-1} - \frac{\eta_n}{k}\log\lambda_n + O(1)\right\},$$

[3] A denotes a positive constant.

and the last expression tends to infinity with n. Hence

$$\limsup_{r\to\infty} \frac{\log M(\tfrac{3}{2}r, F)}{n(r, F)\psi(r)} = \infty,$$

and so

$$\limsup_{r\to\infty} \frac{T(r, F)}{\psi(r)n(r, F)} = \infty.$$

Finally, I must thank Dr. T. Vijayaraghavan for helpful criticisms.

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SEPARABILITIES OF ARBITRARY ORDERS AND RELATED PROPERTIES[1]

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For the case of a metric space Gross[1a] established the equivalence of the following to each other and to other properties: (1) each point set is separable; (2) each uncountable point set contains a condensation point of itself.[2] Sierpiński has shown that for a space S Fréchet these two properties are not equivalent; also, he gives properties which are equivalent to (1) or to (2).[3] These properties involve countable collections, and, thus, are concerned with the smallest transfinite cardinal. Appert[4] generalizes by considering extensions to greater cardinals. He shows that certain of these properties imply others, but gives examples to show that they are not equivalent.[5] In a recent paper the author showed the equivalence of the following properties for the case of a space H Fréchet: (3) each uncountable point set contains a limit point of itself; (4) each uncountable point set either contains a condensation point of itself or is separable.[6] This result supplements Sierpiński's work by showing that property (3), which for the case of metric spaces is equivalent to (1) and (2), is a common basis for them in that it necessarily involves elements of either the one or of the other. In Theorem 2 property (3) is shown to be a necessary and sufficient condition for properties which involve for each point set M cardinals "as close as one pleases" to the power of M in

[1] Presented to the Society, December 27, 1939, under the title *Separabilities of higher orders and related properties.*

[1a] Cf. W. Gross, *Zur Theorie der Mengen in denen ein Distanzbegriff definiert ist*, Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, part IIa, vol. 123 (1914), p. 801.

[2] References to contributions by Fréchet may be found in his *Espaces Abstraits et Leur Théorie Considérée Comme Introduction à l'Analyse Générale*, Paris, 1928. The reader is referred to this treatise for definitions.

[3] W. Sierpiński, *Sur l'équivalence de trois propriétés des ensembles abstraits*, Fundamenta Mathematicae, vol. 2 (1921), pp. 179–188. His results hold for more general spaces than the spaces S he mentions.

[4] A. Appert, *Propriétés des Espaces Abstraits les Plus Généraux*, Actualités Scientifiques et Industrielles, nos. 145 and 146, Paris, 1934, pp. 82–88. These extensions are considered in our Theorem 3.

[5] In *Über höherstufige Separabilität und Kompaktheit*, Japanese Journal of Mathematics, vol. 8 (1931), p. 114, Haratomi considers similar properties for the case of a metric space, for which they are equivalent; thus, his work is a generalization of Gross's.

[6] *Concerning limiting sets in abstract spaces* II, Transactions of this Society, vol. 43 (1938), p. 487.

connection with such concepts as separability of higher orders, the density in M of its limit points of higher orders, the extension of the Cantor-Bendixson theorem, and the "almost" perfect compactness of M. In Theorem 1 similar equivalences are established for orders which are not less than an arbitrary regular cardinal α, provided that the following holds: (3′) each point set of power α contains a limit point of itself. Note that if α is the smallest transfinite cardinal, then (3′) is a corollary of (3); and, if (3′) holds for a certain cardinal, it holds for greater cardinals. While our definitions differ from those of Appert, they yield a more systematic theory.

Definitions. If α is a cardinal,[7] the point set M is said to be *strongly α-separable* provided that there exists N such that (1) $\overline{N} \supset M \supset N$, and (2) either N is countable or its power is less than[8] α. If α is the power of M and M is strongly α-separable, we say that M is *semi-separable.* Thus, for a point set whose power α is aleph-one, the properties separability, semi-separability, and strong α-separability are equivalent. If M is a point set, P is a point, and α is the smallest cardinal, β, such that there exists a neighborhood of P, U, for which the power of $U \cdot (M-P)$ is β, then P is a *limit point of M of order α.* Thus, an isolated point of M has order zero; a limit point has an order greater than zero; a condensation point has an order greater than aleph-zero; a complete limit point of M has an order equal to the power of M. In a space H Fréchet a limit point has an infinite order; at the end of this paper we give examples of spaces V Fréchet in which there exist limit points of finite orders. A point of M which has an order less than α is called an *α-isolated point of M.* The point set M is said to be *almost perfectly α-compact in itself* provided that if α and δ are cardinals, $M \supset N$, $\alpha \leqq$ power of N, and $\delta <$ power of N, then there exists in M a limit point of N of an order at least as great as δ. M is *almost perfectly compact in itself* provided that if $M \supset N$ and δ is a transfinite cardinal which is less than the power of N, then M contains a limit point of N of an order at least as great as δ; if, instead, we require that δ be the power of N, then M is perfectly compact in itself. Thus, a countable set is almost perfectly compact in itself. Because of the equivalence of properties (6) and (1) of Theorem 2, if M is almost perfectly compact in itself, the same property holds

[7] In this paper zero and the positive integers are included among the cardinals and the ordinals. Also, according to our usage, finite or vacuous sets are countable. For the properties of transfinite numbers see F. Hausdorff, *Grundzüge der Mengenlehre*, Leipzig, 1914.

[8] If (2) is modified as follows, M is α-separable: Either N is countable or its power is not greater than α; cf. Appert, loc. cit., p. 83.

for each of its subsets; an analogous proposition does not hold for perfectly compact sets. Consider the properties of M: (a) it is perfectly compact in itself; it is almost perfectly α-compact in itself, (b) for α=aleph-zero, or (c) for α=aleph-one; (d) it is almost perfectly compact in itself. Then (c) and (d) are equivalent; for, if N is an uncountable subset of M and $\delta <$ power of N, there exists a transfinite cardinal δ_1 such that $\delta \leqq \delta_1 <$ power of N; the conclusion follows. Each element of the sequence (a), (b), (c) implies any that follows it, but implies none that precede it. However, for a countable set in a space H Fréchet (a) and (b) are equivalent. The following interpretation is suggestive: In (d) the power of N is the "upper bound" of the orders of the limit points of N which belong to M; in (a) this bound is *attained*.

THEOREM 1. *Let α be a regular cardinal which is greater than aleph-zero, T be a space V Fréchet in which the operation of derivation of point sets is distributive,*[9] *and the symbol M denote an arbitrary point set in T. Then the following properties are equivalent.*

(1) *Each point set of power α contains a limit point of itself.*

(2) *Each point set either is strongly α-separable, or it contains a limit point of itself which has an order not less than α.*

(3) *If M has a regular power β and $\alpha \leqq \beta$, then either M is semi-separable or β is the power of each of the following subsets of M:* (a) *that of all non-β-isolated points of M;* (b) *that of all complete limit points of M which belong to M.*

(4) *Each point set is almost perfectly α-compact in itself.*

(5) *Let $\alpha \leqq \beta$.* (a) *The power of the set of all isolated points of M is less than β;* (b) *the power of the set of all β-isolated points of M is not greater than β;* (c) *if $\beta <$ power of M, then M and the set of its non-β-isolated points have the same power.*[10]

PROOF. Clearly, the properties (2) to (5) imply (1). Suppose that E is a point set for which (2) does not hold. Then, for each point P of E there exists a subset $D(P)$ of E such that P is not an element of the closure of $E-D(P)$, $P \in D(P)$, and the power of $D(P)$ is less than α. Let T be a well-ordered sequence of the elements of E and δ

[9] For example, $(E+F)'=E'+F'$; this is equivalent to assuming that Hausdorff's Axioms A and B hold in T. Cf. Fréchet, loc. cit., pp. 172, 173, and 181; and Appert, loc. cit., pp. 23–27. A cardinal α is regular if it is not the sum of fewer than α cardinlas, each of which is distinct from zero and is less than α; cf. W. Sierpiński, *Hypothèse du Continu*, Warsaw and Lwow, 1934, p. 152. With different hypotheses Appert obtains a result similar to part (5); loc. cit., p. 90.

[10] β is not necessarily regular.

be the smallest ordinal in the number class[11] $Z(\alpha)$. We shall define a well-ordered sequence $U=(q_1, q_2, \cdots, q_\omega, q_{\omega+1}, \cdots, q_\beta, \cdots)$ where $\beta<\delta$.

Proceed as follows: Let q_1 be the first element of T. Let β be an ordinal which is greater than unity and is less than δ; suppose that q_x has been defined for all ordinals $x<\beta$, and let U_β be the set of all such q_x's; let $S_\beta=\sum D(q_x)$, for $x<\beta$. Let q_β be the first point of T which does not belong to the closure of S_β.

We shall show that q_β exists for all ordinals $\beta<\delta$. For, if there exist β's for which this is not true, there exists a smallest one, say λ. Then each point of E belongs to the closure of S_λ. Then the power of U_λ and the powers of each of the sets $D(q_x)$ for $x<\lambda$ are each less than α. Since α is regular, the power of S_λ is less than α. Then, in contradiction to our supposition, E is strongly α-separable. Thus, λ does not exist, and the power of U is α.

By definition q_β is not a point or a limit point of $S_\beta \supset U_\beta$. Further, $S_{\beta+1}$, which contains $D(q_\beta)$, contains no point of $U-U_{\beta+1}$; thus q_β is not a limit point of the latter. Since the operation of derivation is distributive and $U=q_\beta+U_\beta+(U-U_{\beta+1})$, it follows that all points of U are isolated in U. This contradicts (1); thus (1) implies (2).

Let K be the set of all non-β-isolated points of M, and suppose that the power of K is less than β. Then the power of $M-K=L$ is β. Since L contains no limit point of itself of an order as great as β, it follows by (2) that there exists a subset H of L which is dense in L and has a power less than β. Then $H+K$ is dense in M and has a power less than β. Thus, M is semi-separable, and (2) implies (3a); (3b) is (3a) for the case $\beta=$ power of M.

Suppose that (4) does not hold. Then there exist two point sets, M and N, and two cardinals, β_1 and ϵ, such that $\alpha\leqq\beta_1=$ power of N, $\epsilon<\beta_1$, $M \supset N$, and N has no limit point in M of an order as great as ϵ. Let ϵ_1 be the smallest cardinal which is greater than ϵ, β be the larger of α and ϵ_1, and δ be the smallest ordinal in the number class $Z(\beta)$. Then β is regular and does not exceed β_1. Let F be a subset of N having the power β; since $N \supset F$, M contains no limit point of F of an order as great as ϵ. We shall define a well-ordered collection G of subsets of F such that the order type of G is δ: $G=(F_1, F_2, \cdots, F_\omega, \cdots, F_m, \cdots)$, where $m<\delta$.

Proceed as follows: Let T be a well-ordered sequence of all subsets of F, and F_0 be the null set. Suppose that m is an ordinal such that $m<\delta$ and F_x has been defined for all ordinals x which are less than m;

[11] Cf., F. Hausdorff, loc. cit., pp. 124–125.

let $\Sigma_m = \sum F_x$, for $x < m$; let F_m be the first element of T which has a power less than β, which is a subset of $F - \Sigma_m$, and which is dense in $F - \Sigma_m$.

Suppose that the order type of G is λ, where $\lambda < \delta$. Since β is a regular cardinal, it follows that the power of Σ_λ is less than β, and that of $F - \Sigma_\lambda$ is β. Since F contains no limit point of itself of an order as great as ϵ, it follows from (2) that $F - \Sigma_\lambda$ is strongly β-separable. Then F_λ exists; and the supposition that λ exists involves a contradiction.

Let λ be the smallest ordinal in the number class $Z(\epsilon)$, $P \in F_\lambda$, and U be a neighborhood of P. Let x and y be ordinals such that $x < y \leqq \lambda$. Then $\Sigma_y \supset F_x$; since $F - \Sigma_y \supset F_y$, the product $F_x \cdot F_y$ is vacuous. Since $\Sigma_y \supset \Sigma_x$, $F - \Sigma_x \supset F - \Sigma_y \supset F_y$; then F_x is dense on F_y. Thus for $x < \lambda$ the product $U \cdot F_x$ is nonvacuous. Hence, the power of $U \cdot \Sigma_\lambda$ is at least as great as ϵ. Thus, we are involved in a contradiction, and (2) implies (4).

Property (1) implies (5a). Let M_b be the set of all β-isolated points of M, and suppose that its power exceeds β. Let β_1 be the smallest cardinal which is greater than β. Then β_1 is regular, and by (4) each subset of M_b having power β_1 contains a limit point of itself of order at least as great as β. Thus, we are involved in a contradiction; and (4) implies (5b), which implies (5c). Since (1) and (4) are equivalent, (4) implies (5).

THEOREM 2. *In a space V Fréchet in which the operation of derivation of point sets is distributive, the following are equivalent:*

(1) *Each uncountable set contains a limit point of itself.*

(2) *Each point set either contains a condensation point of itself, or it is separable.*

(3) *Each point set which has a regular power either is semi-separable, or it contains a complete limit point of itself.*

(4) *Each point set either is separable, or it has the same power as the set of all its condensation points which belong to it.*

(5) *Each point set which has a regular power either is semi-separable or it has the same power as the set of all its complete limit points which belong to it.*

(6) *Each point set is almost perfectly compact in itself.*

(7) *Part* (5) *of Theorem* 1 *holds for each transfinite cardinal* β *such that aleph-zero* $< \beta$.

PROOF. This may be established by letting the α of Theorem 1 be aleph-one. In particular, to establish (4) apply the following parts of Theorem 1: (a) Part (3b), if aleph-one = power of M; (b) part (5c), if aleph-one < power of M.

THEOREM 2A. *The following modification of Theorem* 1 *is permissible*: (I) *Allow* α *to be any regular cardinal which is greater than unity*; (II) *replace the property "*M *is strongly* α*-separable" by the following*: *"there exists* N *such that* $\overline{N} \supset M \supset N$ *and that the power of* N *is less than* α*."*

Note that the only finite cardinal which satisfies (I) is $\alpha = 2$. Except for obvious modifications, the proof is like that of Theorem 1. Note, however, the following: In considering the proof for (3) for the case power of $M = 2$, let $P \in M \cdot M'$; then $M - P$ is dense in M.

The following theorem gives relations between Appert's work and ours.

THEOREM 3. *Let* β *be transfinite,* α *be the cardinal which is next greatest to* β*, and* T *be a space* V *Fréchet. Each of the following implies property* (1) *of Theorem* 1.[12]

(1) T *is* β*-perfectly separable.*

(2) *Each point set in* T *is* β*-separable.*

(3) *Each point set in* T *possesses the Lindelöf property of order* β.

(4) *Each point set of* T *is* β*-condensed in itself.*[13]

Comment. If M is a point set in the space T of Theorem 1 and $\alpha \leqq \lambda <$ power of M, then by (4) of that theorem M contains a limit point of itself of an order at least as great as λ; that is, the upper bounds of the aggregates of the orders of the limit points which are involved in property (1) are the powers of the sets M concerned. Further, if (1) holds for a given cardinal, it and the properties equivalent to it hold for all greater cardinals. If α is an arbitrary cardinal which is greater than unity, whether finite or transfinite, the following example establishes the existence of a space T such that α is the smallest cardinal for which (1) holds relative to T.

Let α and β be cardinals such that $\alpha < \beta$; let $S_{\alpha\beta}$ be a space whose points are the elements of an aggregate, of power β, and in which the set U is a neighborhood of the point P if and only if the following conditions hold: (1) $P \in U$; (2) the power of the complement of U is

[12] The converse is not true. Note that in Theorem 1 we have the additional hypothesis that the operation of derivation of point sets be distributive.

[13] Property (1) means that T has a basis of a power that does not exceed β. Property (3) means that if a collection G of sets covers the point set M, then there exists H such that $G \supset H$, H covers M, and the power of H does not exceed β. The set M is β-condensed in itself provided that if α is the smallest cardinal which is greater than β, $M \supset K$ and power of $K = \alpha$, then there exists a point P of M such that if U is a neighborhood of P then α is the power of the product of K and the interior of U. Cf. Appert, loc. cit., pp. 82–88.

less than α, and is zero if α is finite. The following are properties of this space: (A) if $\alpha \geqq$ aleph-one, no point set of power α or greater is strongly α-separable; (B) if $\alpha < \lambda$ and power $M = \lambda$, then M is strongly λ-separable; (C) if $\alpha \leqq$ power M, then each point of M is a complete limit point of M; thus property (1) of Theorem 1 holds; (D) if α is the smallest transfinite cardinal, the space is perfectly compact; (E) if α is transfinite, the space is a space H Fréchet; (F) the space satisfies the hypothesis of Theorem 1, for α any cardinal.[14] To establish (A) let power $M \geqq \alpha$, $M \supset N$, power $N < \alpha$, $P \varepsilon (M - N)$; then $P + (S_{\alpha\beta} - N)$ is a neighborhood of P, and P is not a limit point of N. To establish (B) let $M \supset N$, $\alpha <$ power M, $\alpha \leqq$ power $N <$ power M, $P \varepsilon (M - N)$, and U be a neighborhood of P; then power of $N \cdot (S_{\alpha\beta} - U) < \alpha$, and $N \cdot U$ is nonvacuous; thus P is a limit point of N and N is dense in M. A consideration of the property "almost perfect compactness" suggests the question: Does there exist a space in which each infinite point set contains a complete limit point of itself? F. B. Jones gave an answer in the affirmative by suggesting the example in (D). If α is finite, each nonvacuous point set in $S_{\alpha\beta}$ contains a complete limit point of itself.

In Theorem 2, part (2), we consider two alternative properties, of which not more than one is required for each point set. For each of these properties Sierpiński[15] gave an example of a space in which each point set has the given property, but not the alternative property. By taking the sum of his two spaces we have one in which the property that holds for a point set varies with the set. It would be interesting to give an example of a space for which (2) holds and for which each point set that satisfies one of the alternative conditions but not the other has a subset relative to which the converse is true.

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[14] Cf. Appert, loc. cit., p. 25, and Fréchet, loc. cit., p. 186.

[15] Loc. cit., Fundamenta Mathematicae, vol. 2.

CESÀRO SUMMABILITY OF ORDINARY DOUBLE DIRICHLET SERIES

JOHN G. HERRIOT

1. **Introduction.** The purpose of this paper is to obtain some results in the Cesàro summability of ordinary double Dirichlet series similar to those obtained by H. Bohr[1] for the simple series. As is well known a double sequence $\{S_{mn}\}$ may tend to a finite limit as $m, n \to \infty$ [2] without S_{mn} being a bounded function of m and n. In order to avoid difficulties in this respect and to obtain results analogous to those for simple series, the discussion will usually be restricted to bounded sequences.

Let $\sum_{m,n=1}^{\infty} u_{mn}$ be a double series of constant terms. Set

$$
\begin{aligned}
&S_{mn}^{00} = \sum_{i=1,j=1}^{m,n} u_{ij}, \qquad S_{mn}^{10} = \sum_{i=1}^{m} S_{in}^{00}, \qquad S_{mn}^{01} = \sum_{j=1}^{n} S_{mj}^{00}, \\
&S_{mn}^{rs} = \sum_{i=1}^{m} S_{in}^{r-1,s} = \sum_{j=1}^{n} S_{mj}^{r,s-1}, \qquad m, n, r, s = 1, 2, 3, \cdots .
\end{aligned} \tag{1}
$$

The double series is said to be summable (C, r, s) with summability value S if $S_{mn}^{rs} r!s!/m^r n^s \to S$ as $m, n \to \infty$. It is bounded (C, r, s) if $S_{mn}^{rs} r!s!/m^r n^s$ is bounded for all m and n. It is summable-bounded (C, r, s) if it is both summable (C, r, s) and bounded (C, r, s). In case the u_{mn} are functions of complex variables x and y similar definitions can be set up for uniform summability, uniform boundedness, and uniform summability-boundedness (C, r, s).

G. M. Merriman[3] has given the definition of summability (C, r, s) of a double series in a slightly different form. But our means differ from his only by a factor which is bounded and tends to one as $m, n \to \infty$. Consequently the two definitions are equivalent.

By means of Robison's[4] generalization of the Silverman-Toeplitz

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[1] Bohr, H., *Bidrag til de Dirichlet'ske Raekkers Theori*, Dissertation, Copenhagen, 1910; *Über die Summabilität Dirichletscher Reihen*, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, 1909, pp. 247–262; *Sur la série de Dirichlet*, Comptes Rendus de l'Académie des Sciences, Paris, vol. 148 (1909), pp. 75–80.

[2] Throughout this paper $m, n \to \infty$ means m and n tend to infinity simultaneously but independently.

[3] Merriman, G. M., *A set of necessary and sufficient conditions for the Cesàro summability of double series*, Annals of Mathematics, (2), vol. 29 (1928), pp. 343–354.

[4] Robison, G. M., *Divergent double sequences and series*, Transactions of this Society, vol. 28 (1926), pp. 50–73 (p. 53).

theorem, it is easily shown that summability-boundedness $(C, r-1, s)$ (or $(C, r, s-1)$) implies the same (C, r, s).

In §2 of the present paper we give some general lemmas on summable series which are mostly generalizations of theorems of Bohr. These are used in §3 to deduce a number of interesting theorems on summability of ordinary double Dirichlet series. We define associated abscissas of summability-bounded (C, r, s) and in §4 give some theorems showing relations between them.

2. **Preliminary lemmas.** The following lemmas will be useful.

LEMMA 1. *Let $\sum_{m,n=1}^{\infty} u_{mn}(x, y)$ be uniformly summable (C, r, s) for x and y in every D_1' and D_2' respectively for which D_1' and D_2' are closed and contained in the domains D_1 and D_2 respectively, and let $u_{mn}(x, y)$ $(m, n=1, 2, 3, \cdots)$ be regular analytic functions of x and y for x in D_1 and y in D_2. Then the series represents by its summability value $S(x, y)$ an analytic function regular for x in D_1, y in D_2. Moreover the series may be differentiated partially termwise arbitrarily often for x in D_1, y in D_2. The series*

$$\sum_{m=1,n=1}^{\infty,\infty} \frac{\partial^{p+q} u_{mn}(x, y)}{\partial x^p \partial y^q}$$

is summable (C, r, s) for x in D_1, y in D_2, and indeed uniformly for x and y in every D_1' and D_2' respectively, to the function $\partial^{p+q} S(x, y)/\partial x^p \partial y^q$ $(p, q=0, 1, 2, \cdots)$.

LEMMA 2. *Let $F_{mn}(x, y)=\sum_{i=1}^{c} v_i(x)\gamma_{mni}(y)$, where $c=c(m)$ is a function of m defined as a positive integer for positive integral m, and which increases monotonely with m, and let the following four conditions be satisfied:*

(A) $|v_i(x)| \leqq v_i$, *x in domain D_1 $(i=1, 2, 3, \cdots)$;*

(B) $\sum_{i=1}^{\infty} |v_i(x)|$ *converges uniformly in D_1;*

(C) $|\gamma_{mni}(y)| \leqq K$ *(constant), y in domain D_2 $(m, n, i=1, 2, 3, \cdots)$;*

(D) $\lim_{m,n\to\infty} \gamma_{mni}(y)=\gamma_i(y)$ *uniformly in D_2 $(i=1, 2, 3, \cdots)$.*

Then $F_{mn}(x, y)$ is uniformly bounded for x in D_1, y in D_2 for all m and n, and, as $m, n\to\infty$, tends uniformly to the sum of the uniformly absolutely convergent series

$$\sum_{i=1}^{\infty} v_i(x)\gamma_i(y).$$

If the v_i and γ_{mni} are constants, a similar lemma can be stated omitting all references to uniformity and dropping (A).

LEMMA 3. *Let $F_{mn}(x, y)=\sum_{i,j=1}^{c,d} v_i(x)w_j(y)\gamma_{mnij}$, where $c=c(m)$ and*

$d=d(n)$ are functions of m and n, respectively, like $c(m)$ of Lemma 3, and let the following conditions be satisfied:

(A_1) $|v_i(x)| \leqq v_i$, *x in domain* D_1 $(i=1, 2, 3, \cdots)$;

(A_2) $|w_j(y)| \leqq w_j$, *y in domain* D_2 $(j=1, 2, 3, \cdots)$;

(B_1) $\sum_{i=1}^{\infty}|v_i(x)|$ *converges uniformly in* D_1;

(B_2) $\sum_{j=1}^{\infty}|w_j(y)|$ *converges uniformly in* D_2;

(C) $|\gamma_{mnij}| \leqq K$ *(constant)* $(m, n, i, j=1, 2, 3, \cdots)$;

(D) $\lim_{m,n\to\infty}\gamma_{mnij}=\gamma_{ij}$ $(i, j=1, 2, 3, \cdots)$.

Then $F_{mn}(x, y)$ is uniformly bounded for x in D_1, y in D_2 for all m and n, and, as $m, n\to\infty$, tends uniformly to the sum of the uniformly absolutely convergent series $\sum_{i,j=1}^{\infty}v_i(x)w_j(y)\gamma_{ij}$.

Again if the v_i and w_j are constants a similar lemma can be stated omitting all references to uniformity and dropping (A).

We may omit the proofs of these lemmas since they are analogous to those of Bohr.[5]

LEMMA 4. *Let $\sum_{m,n=1}^{\infty}u_{mn}$ be bounded (C, r, s) and let $\{\alpha_m(x)\}$ and $\{\beta_n(y)\}$ be sequences of functions of the complex variables x and y which satisfy the conditions:*

(A_1) $|\alpha_m(x)| \leqq K_m$, *x in domain* D_1 $(m=1, 2, 3, \cdots)$;

(A_2) $|\beta_n(y)| \leqq K_n$, *y in domain* D_2 $(n=1, 2, 3, \cdots)$;

(B_1) $\sum_{m=1}^{\infty}m^{p-1}|\Delta^p\alpha_m(x)|$ *converges uniformly*[6] *in* D_1 $(p=1, 2, 3, \cdots, r+1)$;

(B_2) $\sum_{n=1}^{\infty}n^{q-1}|\Delta^q\beta_n(y)|$ *converges uniformly in* D_2 $(q=1, 2, 3, \cdots, s+1)$;

(C_1) $\lim_{m\to\infty}\alpha_m(x)=0$, *$x$ in* D_1;

(C_2) $\lim_{n\to\infty}\beta_n(y)=0$, *$y$ in* D_2.

Then the series $\sum_{m,n=1}^{\infty}u_{mn}\alpha_m(x)\beta_n(y)$ is uniformly summable-bounded (C, r, s) for x in D_1, y in D_2.

If the α_m and β_n are constants, a similar lemma can be stated omitting all references to uniformity and dropping (A).

For simplicity of notation we shall indicate the proof of this simpler lemma. Let T_{mn}^{rs} be formed from $\sum_{m,n=1}^{\infty}u_{mn}\alpha_m\beta_n$, S_{mn}^{rs} from $\sum_{m,n=1}^{\infty}u_{mn}$. First we obtain the following relation between T_{mn}^{rs} and S_{mn}^{rs}:

$$T_{mn}^{rs}=\sum_{p=0,q=0}^{r+1,s+1}C_{r+1,p}C_{s+1,q}P_{r,s,p,q,m,n},\tag{2}$$

[7]

where

[5] Bohr, loc. cit., Dissertation, pp. 53–60.

[6] $\Delta^p\alpha_m=\alpha_m-p\alpha_{m+1}+\frac{1}{2}p(p-1)\alpha_{m+2}-\cdots+(-1)^p\alpha_{m+p}$.

[7] $C_{r,p}$ denotes the binomial coefficient $r!/p!(r-p)!$.

$$
\begin{aligned}
P_{r,s,0,0,m,n} &= S^{rs}_{mn}\alpha_m\beta_n, \\
P_{r,s,p,0,m,n} &= \sum_{i=1}^{m-p} S^{rs}_{in}C_{m-i-1,p-1}\Delta^p\alpha_i\beta_n, \quad p = 1, 2, 3, \cdots, r+1, \\
P_{r,s,0,q,m,n} &= \sum_{j=1}^{n-q} S^{rs}_{mj}C_{n-j-1,q-1}\alpha_m\Delta^q\beta_j, \quad q = 1, 2, 3, \cdots, s+1, \\
P_{r,s,p,q,m,n} &= \sum_{i=1,j=1}^{m-p,n-q} S^{rs}_{ij}C_{m-i-1,p-1}C_{n-j-1,q-1}\Delta^p\alpha_i\Delta^q\beta_j, \\
&\qquad p = 1, 2, 3, \cdots, r+1; \; q = 1, 2, 3, \cdots, s+1;
\end{aligned}
\tag{3}
$$

if $m \leqq p$ or $n \leqq q$ or both, then $P_{r,s,p,q,m,n} = 0$.

The proof is by induction on r and s successively making use of the relation (for induction on r)

$$
\sum_{i=p+1}^{m} P_{r,s,p,q,i,n} = P_{r+1,s,p,q,m,n} + P_{r+1,s,p+1,q,m,n}. \tag{4}
$$

Next we evaluate $\lim_{m,n\to\infty} T^{rs}_{mn}r!s!/m^rn^s$. By hypothesis

$$
\left| S^{rs}_{mn}r!s!/m^rn^s \right| \leqq K, \qquad m, n = 1, 2, 3, \cdots.
$$

Considering separately the various terms of (2) multiplied by $r!s!/m^rn^s$, we can show that all terms except that for which $p=r+1$, $q=s+1$ tend to zero and are bounded. For example, in case $p=1, 2, 3, \cdots, r+1$, and $q=0$, we use Lemma 2 (modified) choosing

$$
c = m-p, \; v_i = i^{p-1}\Delta^p\alpha_i
$$

and taking up the remaining factors in γ_{mni}. Then $\gamma_{mni}\to 0$ as $m, n\to\infty$ and conditions (B), (C), and (D) of Lemma 2 are satisfied. In case $p=1, 2, 3, \cdots, r+1$ and $q=1, 2, 3, \cdots, s+1$, we apply Lemma 3 (modified). The term for which $p=r+1$, $q=s+1$ is thus seen to be bounded and to tend to a finite limit not necessarily zero as $m, n\to\infty$. This completes the proof of the (modified) lemma.

3. **Double Dirichlet series.** We now make use of the lemmas of §2 to prove some theorems concerning the summability of the ordinary double Dirichlet series

$$
\sum_{m=1,n=1}^{\infty,\infty} a_{mn}/m^xn^y. \tag{5}
$$

THEOREM 1. *If the double Dirichlet series* (5) *is bounded* (C, r, s) *for*

the place (x_0, y_0), *then it is summable-bounded* (C, r, s) *for every place* (x, y) *for which*[8] $R(x) > R(x_0)$ *and* $R(y) > R(y_0)$.[9]

PROOF. Let $x = x_0 + \delta$, $y = y_0 + \eta$, $R(\delta) > 0$, $R(\eta) > 0$. We apply Lemma 4 (modified) taking

$$u_{mn} = a_{mn}/m^{x_0}n^{y_0}, \quad \alpha_m = 1/m^{\delta}, \quad \beta_n = 1/n^{\eta}, \quad u_{mn}\alpha_m\beta_n = a_{mn}/m^{x}n^{y}.$$

Then $\lim_{m\to\infty} \alpha_m = \lim_{n\to\infty} \beta_n = 0$. Also, using the method of Bohr,[10]

$$\Delta^p\alpha_m = \delta(\delta+1)\cdots(\delta+p-1)\int_m^{m+1} da_1 \int_{a_1}^{a_1+1} da_2 \cdots \cdot \int_{a_{p-2}}^{a_{p-1}+1} da_{p-1} \int_{a_{p-1}}^{a_{p-1}+1} \frac{da_p}{a_p^{p+\delta}}.$$

Hence $m^{p-1}|\Delta^p\alpha_m| \leqq |\delta|\,|\delta+1|\cdots|\delta+p-1| \cdot 1/m^{1+R(\delta)}$ $(p = 1, 2, 3, \cdots, r+1)$, so that condition (B_1) of Lemma 4 is satisfied. Likewise (B_2) is satisfied. By the remark following Lemma 4, Theorem 1 follows.

Even if (5) is summable-bounded (C, r, s) for the place (x_0, y_0), it does not necessarily follow that it is summable-bounded (C, r, s) for every place (x_0, y) for which $R(y) > R(y_0)$, or for every place (x, y_0) for which $R(x) > R(x_0)$. For let $r = s = 0$ and let $a_{11} = -1$, $a_{1n} = 1/n(n-1)$ $(n = 2, 3, 4, \cdots)$, $a_{m1} = 2(-1)^m$ $(m = 2, 3, 4, \cdots)$, $a_{mn} = 2(-1)^{m-1}/n(n-1)$ $(m, n = 2, 3, 4, \cdots)$. Then $S_{mn}^{00} = (-1)^m/n$ $(m, n = 1, 2, 3, \cdots)$. Hence for $x = y = 0$ the series (5) is convergent-bounded with sum zero. But for $x = 0$, $y = 1$ the series fails to converge for

$$T_{mn}^{00} = \sum_{i=1,j=1}^{m,n} \frac{a_{ij}}{i^0 j^1} = (-1)^m \sum_{j=1}^{n-1} \frac{1}{j^2(j+1)} + \frac{(-1)^m}{n^2},$$

which tends to no limit as $m, n \to \infty$.

From Theorem 1 we deduce the existence of associated abscissas of summability-bounded (C, r, s), that is, numbers λ_r, μ_s such that (5) is summable-bounded (C, r, s) in the associated domains $R(x) > \lambda_r$, $R(y) > \mu_s$ and not summable-bounded (C, r, s) in the associated domains $R(x) < \lambda_r$, $R(y) < \mu_s$.

[8] $R(x)$ denotes the real part of x.

[9] This theorem and Theorems 3 and 7 bear some resemblance to but are not the same as Theorem IV of Merriman, *Concerning the summability of double series of a certain type*, Annals of Mathematics, (2), vol. 28 (1927), pp. 515–533. Cf. M. Gurney, *Cesàro summability of double series*, this Bulletin, vol. 38 (1932), pp. 825–827.

[10] Bohr, loc. cit., Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, pp. 248–249.

The question arises whether the series (5) has associated abscissas of summability (C, r, s) (the condition of boundedness not being imposed), that is, whether there exists a pair of numbers λ_r', μ_s' such that the series is summable (C, r, s) at all (x, y) for which $R(x) > \lambda_r'$, $R(y) > \mu_s'$, and is not summable (C, r, s) at any (x, y) for which $R(x) < \lambda_r'$, $R(y) < \mu_s'$. The answer is not in general in the affirmative. The following is an example of a series which is convergent but unbounded at $x = y = 0$, but at all other places it fails to converge and at *all* places it is not summable (C, r, s) for any positive integral r and s. Let $a_{1n} = -a_{2n} = n^n$ $(n = 1, 2, 3, \cdots)$, $a_{m1} = -a_{m2} = m^m$ $(m = 3, 4, 5, \cdots)$, $a_{mn} = 0$ $(m, n = 3, 4, 5, \cdots)$. Then

$$S_{mn}^{00} = (1 - 2^{-z}) \sum_{j=1}^{n} j^{j-y} + (1 - 2^{-y}) \sum_{i=3}^{m} i^{i-z}, \qquad m, n = 3, 4, 5, \cdots.$$

If $x = y = 0$, these S_{mn}^{00} are all 0, but otherwise the required limit does not exist. The evaluation of S_{mn}^{rs} is too involved to give here, but for all $r > 0$, $s > 0$, all (x, y), $S_{mn}^{rs} r!s!/m^r n^s$ tends to no finite limit as $m, n \to \infty$.

That such behavior is exceptional is shown by

THEOREM 2. *If the double Dirichlet series* (5) *is summable* (C, r, s) *in a domain D of the two complex variables x and y, it is bounded* (C, r, s) *at each point of this domain.*

Leja[11] has given a similar theorem concerning the convergence of general double Dirichlet series. Our theorem may be proved by a method analogous to his. The first step is to show that if the series (5) is summable (C, r, s) at the points (x_0, y), where x_0 is fixed and y runs over a plane domain d, then, for $j = 1, 2, 3, \cdots$, the series $\sum_{i=1}^{\infty} a_{ij}/i^{x_0}$ are bounded (C, r).

THEOREM 3. *A double Dirichlet series* (5) *having a pair of associated abscissas of summability-bounded* (C, r, s) λ_r, μ_s *is uniformly summable-bounded* (C, r, s) *for x and y in every pair of associated regions* D_1 *and* D_2 *defined by the relations* D_1: $R(x) \geqq \lambda_r + \epsilon$, $|x| \leqq E$; D_2: $R(y) \geqq \mu_s + \epsilon$, $|y| \leqq E$, ϵ *and E being arbitrary finite positive numbers.*

PROOF. Let $x_0 = \lambda_r + \epsilon/2$, $y_0 = \mu_s + \epsilon/2$, and let x be in D_1, y in D_2. The proof is now similar to that of Theorem 1 except that we apply Lemma 4 as stated.

THEOREM 4. *If* λ_r, μ_s *are a pair of associated abscissas of summabil-*

[11] Leja, F., *Sur les séries de Dirichlet doubles*, Comptes-Rendus du 1mier Congrès des Mathématiciens des Pays Slaves, Warsaw, 1929, pp. 140–158.

ity-bounded (C, r, s) *of the double Dirichlet series*

$$\sum_{m=2,n=2}^{\infty,\infty} a_{mn}/m^x n^y, \tag{6}$$

then λ_r, μ_s *are also a pair of associated abscissas of summability-bounded* (C, r, s) *of*

$$\sum_{m=2,n=2}^{\infty,\infty} a_{mn}(\log m)^\alpha(\log n)^\beta/m^x n^y = \sum_{m=2,n=2}^{\infty,\infty} b_{mn}/m^x n^y, \tag{7}$$

where α *and* β *are arbitrary complex numbers, and conversely.*

PROOF. Clearly it suffices to show that, if (6) is summable-bounded (C, r, s) for (x_0, y_0), then (7) is summable-bounded (C, r, s) at every (x, y) for which $R(x) > R(x_0)$ and $R(y) > R(y_0)$. To prove this we take $x = x_0 + \delta$, $y = y_0 + \eta$, $R(\delta) > 0$, $R(\eta) > 0$, and apply Lemma 4 (modified) with

$$u_{mn} = a_{mn}/m^{x_0} n^{y_0}, \qquad \alpha_m = (\log m)^\alpha/m^\delta, \qquad \beta_n = (\log n)^\beta/n^\eta,$$
$$u_{mn}\alpha_m\beta_n = b_{mn}/m^x n^y.$$

Then $\sum_{m,n=2}^{\infty} u_{mn}$ is summable-bounded (C, r, s) and condition (C) is satisfied. Again, using the method of Bohr[12]

$$\Delta^p \alpha_m = (-1)^p \int_m^{m+1} da_1 \int_{a_1}^{a_1+1} da_2 \cdots \cdot \int_{a_{p-2}}^{a_{p-2}+1} da_{p-1} \int_{a_{p-1}}^{a_{p-1}+1} \frac{d^p}{da_p^p} \frac{(\log a_p)^\alpha}{a_p^\delta}\, da_p,$$
$$p = 1, 2, 3, \cdots, r+1.$$

Now the integrand in the last integral is easily shown to be the sum of a finite number of terms of the form a constant times $(\log a_p)^\gamma/a_p^{p+\delta}$. Also $(\log a_p)^\gamma/a_p^{R(\delta)/2}$ is bounded for $a_p \geqq 2$. Hence $m^{p-1}|\Delta^p \alpha_m|$ is not greater than a constant times $1/m^{1+R(\delta)/2}$, so that condition (B_1) and similarly (B_2) of Lemma 4 are satisfied and the desired result follows at once.

THEOREM 5. *A series* (5) *having a pair of associated abscissas of summability-bounded* (C, r, s) λ_r, μ_s *represents by its summability value an analytic function* $f(x, y)$ *regular in the associated half-planes* $R(x) > \lambda_r$, $R(y) > \mu_s$. *The series may be differentiated partially termwise arbitrarily often in these half-planes, that is, the double Dirichlet series*

[12] Bohr, loc. cit., Dissertation, p. 83.

$$\sum_{m=1,n=1}^{\infty,\infty} a_{mn}(-\log m)^p(-\log n)^q/m^x n^y$$

is again summable-bounded (C, r, s) *in the same associated half-planes to the function* $\partial^{p+q}f(x, y)/\partial x^p \partial y^q$ $(p, q = 0, 1, 2, \cdots)$.

This theorem follows immediately from Theorems 3 and 4 and Lemma 1.

A theorem similar to Theorem 5 can be stated and proved for termwise integration of (5) within its domain of summability-bounded.

THEOREM 6. *The analytic function defined by the summability value* (C, r, s) *of* (5) *forms the analytic continuation of the analytic function defined by the sum of the series in a pair of associated half-planes,* $R(x) > \lambda_0$, $R(y) > \mu_0$, *of convergence-bounded (provided that* λ_0, μ_0 *are not also associated abscissas of summability-bounded* (C, r, s)*).*

This is clear from the consistency of the Cesàro mean transformation.

THEOREM 7. *If the series* (5) *is bounded* $(C, r+1, s)$ *for* (x_0, y_0), *then it is summable-bounded* (C, r, s) *for every* (x, y) *for which* $R(x) > R(x_0)+1$ *and* $R(y) > R(y_0)$.

PROOF. Let $x = x_0+1+\delta$, $y = y_0+\eta$, $R(\delta) > 0$, $R(\eta) > 0$. Set

$$u_{mn} = a_{mn}/m^{x_0}n^{y_0}, \quad \alpha_m = 1/m^{1+\delta}, \quad \beta_n = 1/n^{\eta}, \quad u_{mn}\alpha_m\beta_n = a_{mn}/m^x n^y.$$

Let S_{mn}^{rs} be formed from $\sum_{m,n=1}^{\infty} u_{mn}$, T_{mn}^{rs} from (5). Then

$$|S_{mn}^{r+1,s}(r+1)!s!/m^{r+1}n^s| \leqq K, \qquad m, n = 1, 2, 3, \cdots.$$

Since $S_{mn}^{rs} = S_{mn}^{r+1,s} - S_{m-1,n}^{r+1,s}$, we have

$$\left|\frac{S_{mn}^{rs}r!s!}{m^{r+1}n^s}\right| \leqq \left|\frac{(|S_{mn}^{r+1,s}| + |S_{m-1,n}^{r+1,s}|)(r+1)!s!}{m^{r+1}n^s(r+1)}\right| \leqq 2K/(r+1),$$

$$m, n = 1, 2, 3, \cdots.$$

We have to prove that $T_{mn}^{rs}r!s!/m^r n^s$ is bounded for all m and n and tends to a finite limit as $m, n \to \infty$. The proof is now similar to that of Lemma 4 except that in applying Lemmas 2 and 3 we take $v_i = i^p\Delta^p\alpha_i$ instead of $i^{p-1}\Delta^p\alpha_i$.

COROLLARY. *If* λ_r, μ_s *are a pair of associated abscissas of summability-bounded* (C, r, s) *of* (5), *there exists a pair of associated abscissas of summability-bounded* $(C, r+1, s)$ λ_{r+1}, μ_s *such that* $0 \leqq \lambda_r - \lambda_{r+1} \leqq 1$.

4. **Associated abscissas of summability-bounded.** Leja[13] has given a number of theorems showing the relationship between the associated abscissas of convergence-bounded of general double Dirichlet series. We now generalize the content of these theorems for the case of ordinary double Dirichlet series to obtain relations between the associated abscissas of summability-bounded (C, r, s) of the series (5). For simplicity of notation we set

$$A = \limsup_{m+n\to\infty} \frac{\log \left| S_{mn}^{rs} r!s!/m^r n^s \right|}{\lambda_r \log m + \mu_s \log n},$$

where S_{mn}^{rs} is formed from $\sum_{m,n=1}^{\infty} a_{mn}$.

THEOREM 8. *If $\lambda_r > 0$ and $\mu_s > 0$ and if $A \leqq 1$, then the series (5) is summable-bounded (C, r, s) for every (x, y) for which $R(x) > \lambda_r$ and $R(y) > \mu_s$.*

PROOF. Let $x = \lambda_r + \delta$, $y = \mu_s + \eta$, $R(\delta) > 0$, $R(\eta) > 0$. Choose $\epsilon > 0$ but less than the minimum of $R(\delta)/2\lambda_r$ and $R(\eta)/2\mu_s$. Then there exists $K(\epsilon)$ such that

$$\left| S_{mn}^{rs} r!s!/m^r n^s \right| < K m^{\lambda_r(1+\epsilon)} n^{\mu_s(1+\epsilon)}, \qquad m, n = 1, 2, 3, \cdots.$$

Form T_{mn}^{rs} from (5) and set

$$u_{mn} = a_{mn}, \qquad \alpha_m = 1/m^x, \qquad \beta_n = 1/n^y, \qquad u_{mn}\alpha_m\beta_n = a_{mn}/m^x n^y.$$

It suffices to prove $T_{mn}^{rs} r!s!/m^r n^s$ is bounded for all m and n and tends to a finite limit as $m, n \to \infty$. The proof is now similar to that of Lemma 4 except that in applying Lemmas 2 and 3 we take $v_i = i^{p-1}\Delta^p \alpha_i \, i^{\lambda_r + R(\delta)/2}$ instead of $i^{p-1}\Delta^p \alpha_i$.

THEOREM 9. *If $\lambda_r > 0$ and $\mu_s > 0$ and if the series (5) is summable-bounded (C, r, s) for all (x, y) for which $R(x) > \lambda_r$ and $R(y) > \mu_s$, then $A \leqq 1$.*

PROOF. Let any $\epsilon > 0$ be assigned. Then (5) is summable-bounded (C, r, s) for $x = \lambda_r(1+\epsilon/2)$, $y = \mu_s(1+\epsilon/2)$. Set

$$u_{mn} = a_{mn}/m^x n^y, \qquad \alpha_m = m^x, \qquad \beta_n = n^y, \qquad u_{mn}\alpha_m\beta_n = a_{mn}.$$

Forming S_{mn}^{rs} from $\sum_{m,n=1}^{\infty} a_{mn}$ and T_{mn}^{rs} from $\sum_{m,n=1}^{\infty} u_{mn}$ and interchanging T_{mn}^{rs} and S_{mn}^{rs} in (2) we obtain finally

$$\left| \frac{S_{mn}^{rs} r!s!}{m^r n^s} \right| \leqq \frac{r!s!}{m^r n^s} \sum_{p=0,q=0}^{r+1,s+1} C_{r+1,p} C_{s+1,q} \left| P_{r,s,p,q,m,n} \right|, \tag{8}$$

[13] Leja, F., loc. cit.

where now the $P_{r,s,p,q,m,n}$ involve the T_{ij}^{rs} in place of the S_{ij}^{rs}. Since $|T_{mn}^{rs} r!s!/m^r n^s| \leqq K$ (constant) $(m, n = 1, 2, 3, \cdots)$, it is easily shown that each term of the right member of (8) is dominated by $K' m^x n^y$.[14] Hence

$$|S_{mn}^{rs} r!s!/m^r n^s| \leqq K'' m^x n^y = K'' m^{\lambda_r(1+\epsilon/2)} n^{\mu_s(1+\epsilon/2)}$$

Consequently there exists N such that for all m, n for which $m+n>N$

$$|S_{mn}^{rs} r!s!/m^r n^s| \leqq m^{\lambda_r(1+\epsilon)} n^{\mu_s(1+\epsilon)},$$

from which the theorem follows.

THEOREM 10. *A necessary and sufficient condition that* λ_r, μ_s (>0) *be a pair of associated abscissas of summability-bounded* (C, r, s) *of the series* (5) *is that* $A = 1$.

This theorem is an immediate consequence of Theorems 8 and 9.

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[14] The proof of this is analogous to that of Bohr, loc. cit., Dissertation, pp. 92–93.

A CHARACTERIZATION OF EUCLIDEAN SPACES

R. S. PHILLIPS

The purpose of this paper is to give an elementary proof of the fact that a Banach space in which there exist projection transformations of norm one on every two-dimensional linear subspace is a euclidean space. S. Kakutani [1] has pointed out that a modification of a proof due to Blaschke [2] will prove this theorem. F. Bohnenblust has been able to establish this theorem for the complex case by still another method.[1]

A Banach space is a linear, normed, complete space [3, chap. 5]. A euclidean space of dimension α, where α is any cardinal number, is defined to be the Banach space of sequences x, of real numbers where ν ranges over a class of cardinal number α, and $\sum x_\nu^2$ is finite and equal to the square of the norm [4]. We consider only spaces having at least three linearly independent elements.

P. Jordan and J. von Neumann have shown [5] that a Banach space which is euclidean in every two-dimensional linear subspace is itself a euclidean space. It is thus sufficient to show that the "unit sphere" S for any three-dimensional linear subspace is an ellipsoid.

Because of the norm properties, S is a convex body symmetric about the origin o, and contains o as an interior point. Let γ be a plane containing o and let C_γ be the curve of intersection of γ and the boundary S' of S. The existence for each γ of a projection operation of norm one, whose direction of projection is that of the unit vector v_γ, implies that the cylinder generated by lines of direction v_γ tracing C_γ contains S. Our theorem is therefore an immediate consequence of the following lemma on convex bodies (which need not be symmetric about o).

LEMMA.[2] *If S is a convex body such that for every γ there exists a cylinder generated by C_γ containing S, then S is an ellipsoid.*

We topologize the planes γ by representing each by its direction cosines as a point on the unit sphere and using the usual topology of the unit sphere.

The proof of the lemma is divided into two parts. We first show that v_γ is uniquely determined by γ, that v_γ is a continuous function of γ, and that S' has a tangent plane at each of its points. It is then

[1] F. Bohnenblust's result is not yet published.

[2] W. Blaschke has proved a similar theorem under the assumption that there exists a tangent plane at each point of S' [2].

easy to demonstrate that the curves of intersection for any set of parallel planes are similar. Finally we prove that such a convex body is an ellipsoid.

We state without proof the following elementary propositions about boundaries of convex bodies. S' and C_γ are homeomorphs of the two-sphere and the one-sphere respectively. At each point of C_γ there exist two one sided tangents. The right (or left) sided tangents at points q approach the right (or left) sided tangents at p as q approaches p from the right (or left). Furthermore C_γ has a tangent at all except a denumerable set of its points. Given any one sided tangent t at p to C_γ, there exists a plane of support at p containing[3] t. Finally given a tangent t at p to C_γ there exist two half tangent planes to S' at p each containing t; that is, for any $\beta \neq \gamma$ containing the line $\overline{op}$, each one sided half tangent to C_β at p is contained in one of two half-planes, whose common bound is[4] t.

Suppose C_γ has a tangent t at p and that P_1, P_2 are the half tangent planes at p to S'. P_1 and P_2 may or may not be distinct. Let $d(x)$ be the distance from $x \in S'$ to the closest of P_1 and P_2, and let $r(x)$ be the distance from $x \in S'$ to p. The convexity of S then implies that $d(x) \leqq e(r) \cdot r(x)$ where $e(r) \to 0$ with r. For a $\beta \neq \gamma$ containing the line $\overline{op}$, we consider any $x \in C_\beta$ and denote by P_x the plane P_1 or P_2 from which $d(x)$ is measured. Then in order that the line through x in the direction of v_β does not pierce S', it is necessary that the component of v_β perpendicular to P_x be less than $2 \cdot e(2r(x))/\sin(\gamma, \beta)$ which approaches zero as $x \to p$. Hence $p + v_\beta$ is contained in both P_1 and P_2.

It is now easy to show that v_γ is uniquely determined by γ. Suppose two such directions of projection existed and let p be a point of tangency on C_γ with the tangent t. Then there exist two distinct half tangent planes containing t as their common line. The above argument shows that for any β containing $\overline{op}$, $p + v_\beta = t$. But this is impossible for a convex body with interior points.

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[3] Given a one-sided tangent t at p to C_γ, there exists a plane of support at p containing t. C_γ is convex. Therefore C_γ lies entirely on one side of any plane containing t. If the theorem were false, then we could find a plane containing t and interior points of S on both sides of the line through t. The convex extension of some neighborhoods about these points belongs to S and intersects γ on both sides of the line through t, which is impossible.

[4] Given a tangent t at p to C_γ, there exist two half tangent planes at S' at p each containing t. If the contrary were true, there would exist a β containing the line $\overline{op}$ such that a half tangent t' to C_β at p did not determine a plane of support with t at p. The plane through t, t' therefore contains an interior point of S. Again the convex extension of some neighborhood of such a point and C_γ belongs to S and intersects β on both sides of the line through t', which is impossible.

If $v_{\gamma_n} \to v_0$ as $\gamma_n \to \gamma$, then v_0 defines a direction of projection for γ. If the contrary were true, there would exist $p \in C_\gamma$ and a real number λ such that $q = p + \lambda v_0$ is an interior point of S. Let β be the plane determined by o, p, q and p_n the point of $C_{\gamma_n} \cdot C_\beta$ closest to p. Then, as $n \to \infty$, $p_n \to p$ and the points not interior to S, $p_n + \lambda v_{\gamma_n} \to q$, which is impossible. By the uniqueness, $v_0 = v_\gamma$. Since the v_γ form a compact set, it follows that v_γ is a continuous function of γ.

We next show that S' has a tangent plane at each of its points. Let $p \in S'$, γ contain $\overline{op}$, and let t be, say, the right sided tangent at p to C_γ. Then there exist points $p_n \in C_\gamma$ which approach p from the right and which have tangents t_n to C_γ. Let us choose a subsequence for which a set of half tangent planes $P_{n'}$ converge. For convenience we renumber this subsequence 1, 2, $\cdots$, n, $\cdots$. Suppose $P_n \to P$. Then P contains t. Finally for any $\beta \neq \gamma$ containing $\overline{op}$, let us choose a sequence β_n containing $\overline{op_n}$ such that $\beta_n \to \beta$. As above, $p_n + v_{\beta_n}$ lies in P_n and since $v_{\beta_n} \to v_\beta$, $p + v_\beta$ lies in P. As v_β does not lie in β, it follows that the v_β for β containing $\overline{op}$ determine P. But t was an arbitrary one-sided tangent at p. Hence P contains all one sided tangents to curves C_β (β containing $\overline{op}$) and is therefore the tangent plane to S' at p.

We now define any directed line through o to be the z axis. The x-y plane is then the plane containing o which is parallel to the tangent plane to S' at the intersection p of S' and the z axis. In a system of cylindrical coordinates, let γ_θ be the plane $\theta = \text{const}$. Then v_{γ_θ} lies in the x-y plane. The curve of intersection C_z of S' with the plane $z = \text{const.}$ is defined by the differential equation

$$dr/d\theta = rF(\theta), \qquad r(0) = f(z)$$

where $F(\theta)$ is a continuous function independent of z. S' is therefore expressible in the form $r = f(z) \cdot g(\theta)$. Clearly the C_{γ_θ} differ only by a linear transformation.

We next prove that C_γ is an ellipse.[5] For this we need to know that there exists a linear orientation-preserving transformation sending C_{γ_0} into itself and p into any other point q of C_{γ_0}.

Let r be the point of tangency of a plane parallel to the x-z plane having a positive y component. Suppose γ_1 is the plane defined by p, o, r. We have shown that C_{γ_0} goes into C_{γ_1} by a linear transformation which leaves invariant points of the z axis. We can repeat the above construction about the line $\overline{or}$. Hence if γ_2 is the plane defined

[5] The remainder of the proof is similar to an argument used by Garrett Birkhoff, Duke Mathematical Journal, vol. 1 (1935), pp. 169–172, Theorem 1.

by q, o, r, then C_{γ_1} goes into C_{γ_2} and p goes into q by a linear transformation leaving the x-z plane and points of $\overline{or}$ invariant. Repeating the above construction about the line $\overline{oq}$, C_{γ_2} goes into C_{γ_0} by a linear transformation which leaves points of $\overline{oq}$ invariant. The product of these transformations is the desired linear transformation.

The set C_{γ_0} is compact and bounded away from o. Therefore the group of all orientation-preserving linear transformations of C_{γ_0} into itself is bounded and hence equivalent, after a linear transformation, to a subgroup G of the orthogonal group [6, p. 465, Theorem 19]. Since G is transitive on lines through o, G must be the entire orthogonal group. The set of points invariant under G is the circle. Therefore a suitable linear transformation sends C_{γ_0} into the circle. It follows that all C_γ through p are ellipses. p was chosen arbitrarily. All C_γ are therefore ellipses. If we now take a particular C_γ and choose its major axis to be the z axis of our construction, S' will be generated by this ellipse tracing an ellipse in the x-y plane and rotating about the z axis. S is therefore an ellipsoid.

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METRIC SEPARABILITY AND OUTER INTEGRALS[1]

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R. L. Jeffery [1] investigated an upper integral for functions (from the line to real numbers) not necessarily measurable. Let f be bounded, $\alpha<f(x)<\beta$ for $x \in A$, let $e_i=E_x[a_i\leqq f(x)<a_{i+1}]$ where $\alpha=a_0<a_1<\cdots<a_n=\beta$ and consider $\sum a_i m^* e_i$. If as n increases and max $(a_{i+1}-a_i)$ approaches zero, the limit of this sum exists and is independent of the subdivisions, this limit is the upper integral of f over A, $\int_A^* f(x)dx$.

Two point sets with finite outer measure are metrically separable if the outer measure of their sum is the sum of their outer measures. A function is metrically separable on a set A if for each constant λ the part of A on which the function is greater than λ is metrically separable from the rest of A. Jeffery proved that metric separability may be made the basis of a comprehensive theory of integration which includes Young, Pierpont, and Lebesgue integration.

All measurable functions are metrically separable, but a function defined over a non-measurable set (and so necessarily non-measurable) may still be metrically separable. However, if f is metrically separable and possesses an outer integral on a set A, there exists a measurable set $B \supset A$ and a function ϕ, measurable on B and equal to f on A, such that $\int_B \phi = \int_A^* f$. If f is metrically separable and summable on A_1 and on A_2, it need not be metrically separable on A_1+A_2, but is summable on this set. Jeffery's methods of proving these results are not applicable, but the same results hold, as this paper shows, for functions from the plane to real numbers if Carathéodory outer linear measure and integration with respect to this measure are used.[2]

1. **Equivalence.** Let A be a set with finite outer linear (Carathéodory or Gillespie) measure and $\Gamma[A]$ the set of points of the complement of A where the superior outer density of A is positive. Then, [2], $\overline{A}=A+\Gamma[A]$ is a *massgleiche Hülle* of A, that is, is linearly measurable with linear measure equal to the outer linear measure of A. Thus A is linearly measurable if and only if $\Gamma[A]$ has linear measure zero, that is, $L^*\Gamma[A]=0$.

[1] Presented to the Society, in part December 30, 1936, under the title *Metric separability and the Hildebrandt integral*, and in part October 28, 1939, under the title *Metric separability*.

[2] The same methods also prove these results if outer Gillespie linear measure (see [3]) is used.

If L^*A_1 and L^*A_2 are finite, then A_1 and A_2 are metrically separable if and only if

$$LA_1\overline{A}_2 = 0 \qquad (\text{or } LA_2\overline{A}_1 = 0).$$

For $L^*(A_1+A_2) \leqq L^*(A_1+\overline{A}_2) \leqq L(\overline{A_1+A_2}) = L^*(A_1+A_2)$ so the equality holds, and then since $\overline{A}_2$ is measurable

$$L^*(A_1 + \overline{A}_2) = L^*A_1 + L\overline{A}_2 - L^*A_1\overline{A}_2 = L^*A_1 + L^*A_2 - L^*A_1\overline{A}_2,$$

so

$$L^*(A_1 + A_2) = L^*A_1 + L^*A_2$$

if and only if $L^*A_1\overline{A}_2 = 0$. In particular if A_1 and A_2 have no points in common they are metrically separable if and only if

$$L^*A_1\Gamma[A_2] = 0 \qquad (\text{or } L^*A_2\Gamma[A_1] = 0).$$

Also if $A = A_1 + \cdots + A_m$, L^*A_i finite, A_i and A_j metrically separable, then $L^*A = L^*A_1 + \cdots + L^*A_m$ and

$$A + \Gamma[A] = A_1 + \Gamma[A_1] + \cdots + A_m + \Gamma[A_m], \tag{1}$$

both equations following quite simply.

We now prove a theorem for bounded functions on sets of finite outer linear measure that may be extended in the usual way to functions summable, in the sense of outer integrals.

THEOREM. *If A is a plane set with L^*A finite and f is a bounded $(\alpha < f(p) < \beta,\ p \in A)$ function metrically separable on A (so $\int_A^* f$ exists) and $B = A + \Gamma[A]$, then there is a function ϕ which is measurable on B, $p \in A$ implies $\phi(p) = f(p)$, and*

$$\int_B \phi(p)dL(B) = \int_A^* f(p)dL^*(A).$$

If $c < d$ are any two numbers, let $A(c, d) = E_p[c \leqq f(p) < d;\ p \in A]$. For each $n = 1, 2, \cdots$ let $\alpha = a_{n,0}, a_{n,1}, \cdots, a_{n,2^n} = \beta$ be a subdivision such that

$$a_{n,i} - a_{n,i-1} = \beta - \alpha/2^n, \qquad i = 1, 2, \cdots, 2^n,$$

and define

$$A_i = A(a_{n,2^n-i};\ a_{n,2^n-i+1}), \qquad i = 1, 2, \cdots, 2^n.$$

The union of these sets is A and A_i and A_j are metrically separable $i \neq j$ (since f is metrically separable on A); so from (1)

$$B = \sum_{i=1}^{2^n} A_i + \Gamma[A_i].$$

We now define for each $n = 1, 2, 3, \cdots$

$$\phi_n(p) = \begin{cases} f(p) & \text{if } p \in A, \\ a_{n,2^n-i} \text{ if } p \in \{\Gamma[A_i] - (\Gamma[A(a_{n,2^n-i+1})\beta] + A)\}, & i = 1, 2, \cdots. \end{cases}$$

Thus $\phi_n(p)$ is defined for each point p of B. Since $\phi_n(p) \leqq \phi_{n+1}(p) < \beta$ the limit $\phi(p) = \lim_{n\to\infty} \phi_n(p)$ exists. Clearly $\phi(p) = f(p)$ if $p \in A$.

Also ϕ is a measurable function. First $E_p[\phi_n(p) \geqq a_{n,r}] = \overline{A}(a_{n,r}; \beta) - A(\alpha; a_{n,r})$. However, since f is metrically separable on A, the sets $A(a_{n,r}; \beta)$ and $A(\alpha; a_{n,r})$ are metrically separable and have no points in common

$$L\{\Gamma[A(a_{n,r}; \beta)]A(\alpha; a_{n,r})\} = 0,$$

that is, the measurable set $\overline{A}(a_{n,r}; \beta)$ represents almost all points where $\phi_n(p) \geqq a_{n,r}$. Now if λ is any number $\alpha \leqq \lambda \leqq \beta$, let $a_{n_1,r_1}, a_{n_2,r_2}, \cdots$ where $n_1 \leqq n_2 \leqq \cdots$ be an increasing sequence of points of subdivisions approaching λ. Then the measurable set

$$\prod_{i=1}^{\infty} \overline{A}(a_{n_i,r_i}; \beta)$$

differs from $E_p[\phi(p) \geqq \lambda, p \in \beta]$ by a set of measure zero. Thus ϕ is a measurable function.

To prove the integral equality let $\alpha = a_0 < a_1 < \cdots < a_n = \beta$ be a subdivision and define

$$s_i = \underset{p}{E}[p \in A, a_i \leqq f(p) < a_{i+1}], \qquad S_i = \underset{p}{E}[p \in B, a_i \leqq \phi(p) < a_{i+1}].$$

Then S_i is measurable, $S_i \supset s_i$ and

$$L^*A = L^*s_1 + \cdots + L^*s_m \leqq LS_1 + \cdots + LS_m = LB = L^*A,$$

so $L^*s_i = LS_i$, $i = 1, 2, \cdots, m$, and thus by multiplying by a_i, summing and taking limits we see the desired equality

$$\int_A^* f = \int_B \phi$$

for bounded functions over sets of finite outer linear measure.

2. **Additivity.** If G_1 and G_2, each with finite outer measure, are metrically separable and f is a function metrically separable on each, f is metrically separable on $G_1 + G_2$. However, if G_1 and G_2 are not

metrically separable, then f may not be metrically separable on their union, for example, if G_1 and G_2 are complements on the unit interval each with outer measure unity and $f(x)=1$ if $x \in G_1$, $f(x)=2$ if $x \in G_2$. If however, f is metrically separable and the outer integral of f exists on each set G_1 and G_2, then the outer integral exists on G_1+G_2. For let $G_1=G_{11}+G_{12}$ where $G_{12}=G_1(G_2+\Gamma[G_2])$ and $G_{11}=G_1-G_{12}$ and in like manner take $G_2=G_{21}+G_{22}$. Then G_{11} and G_{12} are metrically separable as are G_{21} and G_{22} as well as G_{11} and G_{21}, while no subset of positive outer measure of G_{12} is metrically separable from G_{22} and no subset of G_{22} is metrically separable from G_{12}. We may then apply the following theorem (proved for positive bounded functions, but extendible in the usual way) to obtain the property of additivity over sets of the existence of outer integrals.

THEOREM. *Let A_1 and A_2 be two point sets of the plane with outer linear measure finite such that no part of either with positive outer linear measure is metrically separable from the other. Then if f is a positive and bounded $(0<f(p)<\beta;\ p \in A_1+A_2)$ function metrically separable on A_1 and on A_2, the outer integral of f exists on A_1+A_2.*

For each $n=1, 2, 3, \cdots$ let $0=a_{n,0}, a_{n,1}, \cdots, a_{n,2^n}=\beta$ be a subdivision of the interval $(0, \beta)$ into 2^n equal parts. Define

$$s_{n1i} = \underset{p}{E}[p \in A_1,\ a_{n,i-1} \leqq f(p) < a_{n,i}],$$

$$s_{n2i} = \underset{p}{E}[p \in A_2,\ a_{n,i-1} \leqq f(p) < a_{n,i}], \qquad i = 1, 2, \cdots, 2^n,$$

and

$$e_n = \sum_{i=1}^{2^n} \overline{s_{n1i}}\ \overline{s_{n2i}}$$

where $\overline{s_{n1i}}=s_{n1i}+\Gamma[s_{n1i}]$. Then e_n is a measurable subset of the measurable set $\overline{A_1}+\overline{A_2}$. Also let

$$E_n = (\overline{A_1}+\overline{A_2}) - e_n.$$

Since $s_{n+1,1,2i-1}$ and $s_{n+1,1,2i}$ are metrically separable and their union is s_{n1i} we have

$$\bar{s}_{n+1,1,2i-1} + \bar{s}_{n+1,1,2i} = \bar{s}_{n,1,i}$$

and

$$\bar{s}_{n+1,2,2i-1} + \bar{s}_{n+1,2,2i} = \bar{s}_{n,2,i}, \qquad i = 1, 2, \cdots, 2^n.$$

If then $p \in e_{n+1}$ there will be a j such that p will be either in

$\bar{s}_{n+1,1,2i-1}\bar{s}_{n+1,2,2i-1}$ or $\bar{s}_{n+1,1,2i}\bar{s}_{n+1,2,2i}$ so in either case will belong to $\bar{s}_{n,1,i}\bar{s}_{n,2,i}$ and so to e_n, that is, $e_{n+1} \subset e_n$. Thus $E_{n+1} \supset E_n$ and the sets

$$e = e_1 \cdot e_2 \cdot \cdots, \qquad E = E_1 + E_2 + \cdots$$

are clearly metrically separable.

We first prove that the outer integral of f exists over $e(A_1+A_2)$.

Let $b_{ni} = E_p[p \in e(A_1+A_2),\ a_{n,i-1} \leqq f(p) < a_{n,i}]$, $n=1, 2, 3, \cdots$, $i=1, 2, \cdots, 2^n$. Then $b_{n,i} = e(s_{n,1,i}+s_{n,2,i})$. But $e_n\bar{s}_{n1i}$ and $e\bar{s}_{n2i}$ differ from each other only in a set of measure zero so $e\bar{s}_{n1i}$ and $e\bar{s}_{n2i}$ differ at most by a set of measure zero, so

$$Le(\bar{s}_{n1i} + \bar{s}_{n2i}) = Le\bar{s}_{n1i} = Le\bar{s}_{n2i},$$

so

$$L^*e(s_{n1i} + s_{n2i}) = L^*es_{n1i} = L^*es_{n2i}$$

or

$$L^*b_{ni} = L^*es_{ni} = L^*es_{n2i}.$$

Thus

$$\sum_{i=1}^{2^n} a_{ni}L^*b_{ni} = \sum_{i=1}^{2^n} a_{ni}L^*es_{n1i} = \sum_{i=1}^{2^n} a_{ni}L^*es_{n2i},$$

and consequently the integral of f exists over $e(A_1+A_2)$ and

$$\int_{e(A_1+A_2)}^{*} f(p)dL(p) = \int_{eA_1}^{*} f(p)dL(p) = \int_{eA_2}^{*} f(p)dL(p).$$

We now show that f has an outer integral on $E(A_1+A_2)$ which moreover is the sum of the outer integrals over EA_1 and EA_2.

Let $e_n' = \sum_{i=1}^{2^n} E\bar{s}_{n1i}\bar{s}_{n2i}$ and let $E_n' = E(\overline{A}_1+\overline{A}_2) - e_n'$. Now $e_n' \supset e_{n+1}'$ so $E_n' \subset E_{n+1}'$. Also $\lim_{n\to\infty} L^*e_n' = 0$ for otherwise we would have $L^*Ee > 0$. Therefore $\lim_{n\to\infty} L^*E_n' = E$ and thus $\lim_{n\to\infty} L^*E(A_1+A_2) = L^*E(A_1+A_2)$, $\lim_{n\to\infty} L^*E_n'A_1 = L_1EA_1$ and $\lim_{n\to\infty} L^*E_n'A_2 = EA_2$.

Now for $n' > n$ let $h_{n'ji} = E_p[p \in E_n'A_j,\ a_{n',i-1} \leqq f < a_{n',i}]$, $j=1, 2$; $i=1, 2, \cdots, 2^{n'}$. Then $h_{n'1i}$ and $h_{n'2i}$ are metrically separable, so we have

$$L^*(h_{n'1i} + h_{n'2i}) = L^*h_{n'1i} + L^*h_{n'2i}$$

and thus

$$\int_{E_n'(A_1+A_2)}^{*} f = \int_{E_n'A_1}^{*} f + \int_{E_n'A_2}^{*} f.$$

Also since

$$\int_{E'A_1}^{*} f + \int_{(E-E_n')A_1}^{*} f = \int_{EA_1}^{*} f,$$

we have

$$\lim_{n\to\infty} \int_{E_n'A_1}^{*} f = \int_{EA_1}^{*} f, \qquad \lim_{n\to\infty} \int_{E_n'A_2}^{*} f = \int_{EA_2}^{*} f.$$

Then since

$$\int_{E_n'(A_1+A_2)}^{*} f \leqq \int_{E_{n'+1}(A_1+A_2)}^{*} f \leqq \int_{EA_1}^{*} f + \int_{EA_2}^{*} f,$$

we see that

$$\lim_{n\to\infty} \int_{E_n'(A_1+A_2)}^{*} f \text{ exists}$$

and is less than or equal to

$$\int_{EA_1}^{*} f + \int_{EA_2}^{*} f.$$

We may further assert that the equality holds. For $\epsilon>0$ given there is an N such that $n \geqq N$ implies

$$\int_{E_n'A_j}^{*} f > \int_{EA_j}^{*} f - \frac{\epsilon}{2}, \qquad j = 1, 2,$$

or

$$\int_{E_n'(A_1+A_2)}^{*} f = \int_{E_n'A_1}^{*} f + \int_{E_n'A_2}^{*} f > \int_{EA_1}^{*} f + \int_{EA_2}^{*} f - \epsilon.$$

Thus

$$\int_{A_1+B_1}^{*} f = \int_{E(A_1+A_2)}^{*} f + \int_{e(A_1+A_2)}^{*} f = \int_{EA_1}^{*} f + \int_{EA_2}^{*} f + \int_{e(A_1+A_2)}^{*} f.$$

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A PROPERTY OF A SIMPLY ORDERED SET[1]

K. W. FOLLEY

Sierpinski[2] has shown that the set of real numbers of the interval (0, 1) may be decomposed into c disjoint subsets, each of power less than c, and such that the sum of every c of these subsets has at least one point in common with every perfect subset of the interval.

The object of the present paper is to show that the same method of proof may be used to prove an analogous theorem concerning a more general type of set.

DEFINITIONS. *A simply ordered set M is a set such that if any two of its elements are given it is known which one precedes.*

A subset of M is said to be cofinal (coinitial) with M if no element of M follows (precedes) all the elements of the subset.

An η_α subset of M is one which is neither cofinal nor coinitial with any subset of M of power less than $\aleph_\alpha$ and which contains no pair of neighboring subsets both of which have power less than $\aleph_\alpha$.

Various properties of simply ordered sets M containing everywhere dense η_α subsets, including a discussion of the perfect subsets of M, were discussed by the writer in a previous paper.[3]

THEOREM 1. *Let M be a simply ordered set containing an everywhere dense η_α subset N. There exists a decomposition of M into $2^{\aleph_\alpha}$ disjoint subsets, each of power less than $2^{\aleph_\alpha}$, and such that the sum of every $2^{\aleph_\alpha}$ of these subsets has at least one point in common with every perfect subset of M.*

PROOF. Let ϕ be the smallest ordinal number of power $2^{\aleph_\alpha}$. A transfinite sequence of type ϕ formed of all the points of M exists, namely,

$$m_1, m_2, m_3, \cdots, m_\xi, \cdots, \qquad \xi < \phi. \tag{1}$$

The perfect subsets of M having power $2^{\aleph_\alpha}$ may be arranged in the form of a transfinite sequence of type ϕ as follows:

$$M_1, M_2, M_3, \cdots, M_\xi, \cdots, \qquad \xi < \phi. \tag{2}$$

Let us now define a transfinite sequence $\{H_\xi\}_{\xi<\phi}$ of subsets of M: H_1 is formed of the single element m_1.

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[1] Presented to the Society, November 22, 1940.

[2] Fundamenta Mathematicae, vol. 24 (1935), pp. 8–11.

[3] Proceedings of the Royal Society of Canada, Section III, vol. 22 (1928), pp. 225–239.

Now, let β be any ordinal number between 1 and ϕ. Suppose we have defined all sets H_ξ, where $\xi<\beta$, and suppose further that the power of H_ξ is not greater than $\bar{\xi}$. Let $S_\beta=\sum_{\xi<\beta}H_\xi$. Then S_β is evidently of power not greater than $\bar{\beta}^2<2^{\aleph_\alpha}$.

We shall define a transfinite sequence $\{p_\xi^\beta\}$ as follows. M_1 being of power $2^{\aleph_\alpha}$, and S_β being of power less than $2^{\aleph_\alpha}$, M_1-S_β contains points. Let p_1^β be the first element of (1) which belongs to M_1-S_β. Now let η be any ordinal number between 1 and β, and let us suppose we have already defined the points p_ξ^β, where $\xi<\eta$. Let all these points form the set T_η^β of power not greater than $\bar{\eta}\leqq\bar{\beta}<2^{\aleph_\alpha}$. The set $M_\eta-(S_\beta+T_\eta)$ contains points. Let p_η^β be the first point of (1) which belongs to the set $E_\eta-(S_\beta+T_\eta)$.

Define H_β as the set of all points p_ξ^β, where $\xi<\beta$; it is a set of power not greater than $\bar{\beta}<2^{\aleph_\alpha}$.

The sets H_ξ, with $\xi<\phi$, are thus defined by transfinite induction. They are evidently disjoint subsets of M, each of power less than $2^{\aleph_\alpha}$. The set $R=M-\sum_{\xi<\phi}H_\xi$ is of power not greater than $2^{\aleph_\alpha}$. Hence the elements of R may be arranged in the form of a transfinite sequence of type $\psi\leqq\phi$ as follows:

$$q_1, q_2, q_3, \cdots, q_\xi, \cdots, \qquad \xi<\psi.$$

Let $N_\xi=H_\xi+q_\xi$ for $\xi<\psi$, and if $\psi<\phi$, $M_\xi=H_\xi$ for $\psi\leqq\xi<\phi$.

The sets N_ξ are disjoint subsets of M, each of power less than $2^{\aleph_\alpha}$. Moreover $M=\sum_{\xi<\phi}N_\xi$.

Now, let F be the sum of any $2^{\aleph_\alpha}$ of the sets N_ξ, $\xi<\phi$. If μ is any ordinal number less than ϕ, there is an ordinal number β such that $\mu<\beta<\phi$, and such that N_β belongs to F. Since $\beta>\mu$ we have p_μ^β belonging to H_μ and therefore to N_μ. Besides p_μ^β belongs to M_μ. Hence p_μ^β is a point of $M_\mu\cdot H_\beta$ and so of $F\cdot M_\mu$.

It follows that F has at least one point in common with every subset M_ξ of (2), which was to be proved.

THEOREM 2. *The generalized hypothesis of the continuum* ($2^{\aleph_\alpha}=\aleph_{\alpha+1}$) *is equivalent to the following statement*:

The set M may be decomposed into disjoint subsets, each of power not greater than $\aleph_\alpha$, such that the sum of any class of more than $\aleph_\alpha$ of them has at least one point in common with every perfect subset of M.

PROOF. (a) If the generalized hypothesis of the continuum is assumed, Theorem 1 becomes the second part of Theorem 2.

(b) Suppose that the second part of Theorem 2 is true. There are $2^{\aleph_\alpha}$ perfect subsets of[4] M. Each disjoint subset of M having a power

[4] See K. W. Folley, loc. cit., p. 232.

not greater than $\aleph_\alpha$, it follows that a class of $\aleph_{\alpha+1}$ of them has power $\aleph_{\alpha+1}$. Such a class has at least one point in common with every perfect subset of M. Thus, there results the inequality

$$\aleph_{\alpha+1} \leqq 2^{\aleph_\alpha}.$$

The inequality in the opposite sense being well known, the generalized hypothesis of the continuum follows.

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UNIQUENESS THEOREMS FOR RATIONAL FUNCTIONS[1]

FRITZ HERZOG

In his book on the theory of meromorphic functions,[2] R. Nevanlinna proved a number of "uniqueness theorems." The most important of them[3] states that if two functions $w=f(x)$ and $w=g(x)$, meromorphic in the whole x-plane, assume *five* values of w (finite or infinite) at the same points x they must be identical. If we understand by the *distribution* of a function $w=\phi(x)$ with respect to a given value of w simply the set of all points x where $\phi(x)$ assumes that value w, *regardless of multiplicity*, we may state the above theorem in the following way: Two meromorphic functions which have identical distributions with respect to five values of the dependent variable must be identical. In proving this theorem, Nevanlinna explicitly assumes the functions to be transcendental (i.e., not rational).[3] It is trivial, however, that the theorem would apply to two rational functions $w=f(x)$ and $w=g(x)$ as well, which can be easily seen by considering the transcendental functions $w=f(e^z)$ and $w=g(e^z)$.

The example of the functions $w=e^z$ and $w=e^{-z}$, which have identical distributions with respect to the four values $w=1$, -1, 0, ∞, shows that five is the smallest number for which the above-mentioned uniqueness theorem holds true. It will be shown in this paper that such is not the case for rational functions for which five may, indeed, be replaced by *four*. (See Theorem I.)

The question arises as to what can be said about two rational functions that have identical distributions with respect to only *three*

[1] Presented to the Society, February 24, 1940.

[2] Rolf Nevanlinna, *Le Théorème de Picard-Borel et la Théorie des Fonctions Méromorphes*, Paris, 1929.

[3] See loc. cit., p. 109.

values of the dependent variable.[4] It is the main object of this paper to study this problem.

It will be found that there exist pairs of rational functions which have identical distributions with respect to three values without being identical (Examples I, II, III). Furthermore, under certain restrictive conditions concerning, for instance, the degrees of the functions, uniqueness theorems for rational functions with identical distributions with respect to three values will be derived. (See Theorems II and III.)

Throughout the paper we shall use x or z for the independent complex variable and w for the dependent complex variable. Unless otherwise stated, x is to be restricted to finite values while z and w may assume also the value ∞.

As far as the variables z and w are concerned, we may subject them to linear substitutions without changing the character of the problem under consideration. This is due to the fact that a substitution $u=(au_1+b)/(cu_1+d)$ with $ad-bc\neq 0$ gives a one-to-one correspondence between the u and u_1 planes (both $u=\infty$ and $u_1=\infty$ being included). We shall sometimes make use of such linear substitutions without further explanation.

We finally remark that all functions mentioned in the paper are understood to be not constant, unless otherwise stated.

We consider two rational functions $w=f(x)$ and $w=g(x)$ which have identical distributions with respect to four values of w and shall show that they must be identical. In accordance with what was said above, x is restricted to finite values, that is, the two functions are assumed to have identical distributions with respect to the four values of w, only as far as finite values of x are concerned, while the point $x=\infty$ is disregarded (except at certain points in the proof). We thus obtain a more general theorem.

THEOREM I. *If $w=f(x)$ and $w=g(x)$ are two rational functions and if w_1, w_2, w_3, w_4 are four different values of w that are assumed by the two functions at the same points x, then $f(x)\equiv g(x)$.*

PROOF. Let m and n be the degrees of $f(x)$ and $g(x)$, respectively; we may assume that $m\geqq n$. Let $w_0=f(\infty)$ where w_0 may be either dif-

[4] Cf. Nevanlinna's remark, loc. cit., p. 109: "... une fonction rationnelle est univoquement déterminée par *trois* distributions." In view of the results of this paper, this remark must be understood in such a way that identical distribution is to imply also equal multiplicities. Neither in Nevanlinna's uniqueness theorem nor in this paper is this the case.

ferent from w_1, w_2, w_3, and w_4 or equal to one of them. In the latter case we may assume that $w_0=w_1$.

Now let r_k be the actual number of finite w_k-points of $f(x)$ and $g(x)$ $(k=1, 2, 3, 4)$, that is, their number not counting multiplicity. Let m_0 be the multiplicity of w_0 at $x=\infty$ and m_k be the number of finite w_k-points of $f(x)$ $(k=1, 2, 3, 4)$, counting multiplicity. Then we have $m_1=m$ or $m_0+m_1=m$, according to whether $w_0\neq w_1$ or $w_0=w_1$. Hence, in any case,

$$m_0 + m_1 \geqq m. \tag{1}$$

Also, in any case,

$$m_2 = m_3 = m_4 = m. \tag{2}$$

We may assume, without loss of generality, that w_0, w_1, w_2, w_3, and w_4 are all finite because a suitable linear substitution of w will, if necessary, transform the w_k into finite values.

Now let $f=F_1/F_2$ and $g=G_1/G_2$ be representations of f and g as reduced fractions of polynomials. We consider the functions

$$f' = (F_2F_1' - F_1F_2')/F_2^2, \tag{3}$$

$$f - g = (F_1G_2 - F_2G_1)/F_2G_2. \tag{4}$$

(3) is not identically zero, it is of degree at most $2m$, it vanishes at $x=\infty$ with a multiplicity m_0+1 (which can be seen easily from the expansion of $f(x)$ in a power series about $x=\infty$), and it has m_k-r_k zeros, counting multiplicity, at the r_k finite w_k-points of $f(x)$ $(k=1, 2, 3, 4)$. These facts yield

$$(m_0 + 1) + (m_1 - r_1) + (m_2 - r_2) + (m_3 - r_3) + (m_4 - r_4) \leqq 2m. \tag{5}$$

Using (1) and (2), we obtain from (5)

$$4m + 1 - (r_1 + r_2 + r_3 + r_4) \leqq 2m$$

or

$$r_1 + r_2 + r_3 + r_4 \geqq 2m + 1. \tag{6}$$

On the other hand, (4) is of degree at most $m+n\leqq 2m$, it vanishes at the r_k finite points x where $f(x)=g(x)=w_k$ $(k=1, 2, 3, 4)$, and hence, by (6), it has at least $r_1+r_2+r_3+r_4\geqq 2m+1$ zeros. Therefore, (4) must vanish identically, which proves the theorem.

That Theorem I does not hold when only three identical distributions are assumed can be seen from very simple examples, such as $w=f(x)=4x/(x+1)^2$ and $w=g(x)=2x/(x+1)$ which have identical distributions with respect to the three values $w=0$, ∞, 1, provided

we consider only finite x. The value $w=0$, in particular, is assumed by $f(x)$ and $g(x)$ at $x=0$. If, however, x were allowed to assume also the value ∞, the distributions of $f(x)$ and $g(x)$ with respect to $w=0$ would not be identical any more since $f(\infty)=0$ but $g(\infty)\neq 0$.

We shall, therefore, from now on allow the independent variable to assume also the value ∞ and study the problem of two rational functions $w=f(z)$ and $w=g(z)$ which have identical distributions with respect to three values of w.

We first consider the special case of polynomials. Since a polynomial assumes the value $w=\infty$ at $z=\infty$ only, the question is whether two polynomials with identical distributions with respect to two (finite) values of w must be identical. The answer is given by the following theorem.

THEOREM II. *If two polynomials $w=F(z)$ and $w=G(z)$ assume two finite values w_1 and w_2 $(w_1\neq w_2)$ at the same points z, then $F(z)\equiv G(z)$.*

PROOF. Let m and n be the degrees of $F(z)$ and $G(z)$, respectively, let $m\geqq n$ and let r_k be the actual number of w_k-points of $F(z)$ and $G(z)$ $(k=1, 2)$, not counting multiplicity. Then $F'(z)$ is not identically zero, it is of degree $m-1$ and it has $m-r_k$ zeros, counting multiplicity, at the r_k points where $F(z)=w_k$ $(k=1, 2)$. This gives $(m-r_1)+(m-r_2)\leqq m-1$ or

$$r_1+r_2\geqq m+1. \tag{7}$$

On the other hand, $F(z)-G(z)$ is of degree at most m and it vanishes at the r_k points where $F(z)=G(z)\neq w_k$ $(k=1, 2)$. Hence, by (7), it has at least $r_1+r_2\geqq m+1$ zeros and must, therefore, vanish identically. This proves the theorem.

By subjecting both z and w to linear substitutions the two polynomials become two rational functions which have the property that there is one value of w which is assumed by both functions at one and the same point z only. Thus, according to what was said in the introduction, Theorem II yields immediately the following corollary.

COROLLARY. *If two rational functions $w=f(z)$ and $w=g(z)$ assume three different values w_1, w_2, w_3 at the same points z and if one of these three values is assumed at one point z only, then $f(z)\equiv g(z)$.*

The corollary just proved is one of several similar statements that will be derived in the following. These statements give sufficient conditions under which two rational functions with identical distributions with respect to three values must be identical. It will simplify the language if we put these statements in the form of necessary con-

ditions which two different rational functions have to satisfy if they have identical distributions with respect to three values of the dependent variable.

Thus we consider two rational functions $w=f(z)$ and $w=g(z)$ which assume three different values w_1, w_2, w_3 at the same points z, which we shall call z_{ks} ($k=1, 2, 3$; $s=1, 2, \cdots, r_k$; all z_{ks} different). Furthermore, we assume that the degrees of $f(z)$ and $g(z)$ are m and n, respectively, and that $m \geqq n$. Finally, let m_{ks} and n_{ks} be the multiplicities with which $w=f(z)$ and $w=g(z)$, respectively, assume the value $w=w_k$ at $z=z_{ks}$ ($m_{ks} \geqq 1$, $n_{ks} \geqq 1$). It will be convenient to enter these data in the following table.

Value of w	*Distribution of f(z) and g(z) with respect to w_k*	*Multiplicities at z_{ks} of*	
		$f(z)$	$g(z)$
w_1	$z_{11}, z_{12}, \cdots, z_{1r_1}$	$m_{11}, m_{12}, \cdots, m_{1r_1}$	$n_{11}, n_{12}, \cdots, n_{1r_1}$
w_2	$z_{21}, z_{22}, \cdots, z_{2r_2}$	$m_{21}, m_{22}, \cdots, m_{2r_2}$	$n_{21}, n_{22}, \cdots, n_{2r_2}$
w_3	$z_{31}, z_{32}, \cdots, z_{3r_3}$	$m_{31}, m_{32}, \cdots, m_{3r_3}$	$n_{31}, n_{32}, \cdots, n_{3r_3}$

We note that for $k=1, 2, 3$

$$m_{k1} + m_{k2} + \cdots + m_{kr_k} = m, \tag{8}$$

$$n_{k1} + n_{k2} + \cdots + n_{kr_k} = n. \tag{9}$$

THEOREM III. *If $w=f(z)$ and $w=g(z)$ are two rational functions with identical distributions with respect to three values of w and if w_k, r_k, z_{ks}, m, n, m_{ks}, n_{ks} are as defined above, then the following are necessary conditions for $f(z) \not\equiv g(z)$:*

$$\text{(a)} \qquad \sum_{k=1}^{3} \sum_{s=1}^{r_k} \min (m_{ks}, n_{ks}) \leqq m + n,$$

$$\text{(b)} \qquad r_1 + r_2 + r_3 \geqq m + 2,$$

$$\text{(c)} \qquad 2 \leqq r_k \leqq n, \qquad \textit{for } k = 1, 2, 3,$$

$$\text{(d)} \qquad m + n \geqq 6,$$

$$\text{(e)} \qquad m \leqq 3n - 2,$$

$$\text{(f)} \qquad (m_{k1}, m_{k2}, \cdots, m_{kr_k}) = (n_{k1}, n_{k2}, \cdots, n_{kr_k}) \qquad \textit{for at most one } k.^{5}$$

PROOF. For the proof of conditions (a) to (e) we assume that the three w_k as well as all the z_{ks} are finite numbers.

[5] This equation is to be considered an identity of two vectors, that is, $m_{k1}=n_{k1}$, $m_{k2}=n_{k2}, \cdots, m_{kr}=n_{kr}$.

(a) The function $f(z)-g(z)$ (see (4) above) is not identically zero, it is of degree at most $m+n$, it has a zero at $z=z_{ks}$ the multiplicity of which is at least min (m_{ks}, n_{ks}), and it has, therefore, at least $\sum_{k=1}^{3}\sum_{s=1}^{r_k}$ min (m_{ks}, n_{ks}) zeros, counting multiplicity. Thus (a) must hold.

(b) The function $f'(z)$ (see (3) above) is not identically zero and the numerator $F_2F_1'-F_1F_2'$ is of degree at most $2m-2$. At the points z_{ks} $(s=1, 2, \cdots, r_k)$ $f'(z)$ has $m-r_k$ zeros, counting multiplicities $(k=1, 2, 3)$, hence $f'(z)$ has at least $(m-r_1)+(m-r_2)+(m-r_3)$ finite zeros which must be zeros of the numerator $F_2F_1'-F_1F_2'$. This gives

$$3m-(r_1+r_2+r_3)\leqq 2m-2,$$

from which (b) follows.

(c) The inequality $r_k\geqq 2$ (for $k=1, 2, 3$) is easily seen to be identical with the corollary, proved above. That $r_k\leqq n$ (for $k=1, 2, 3$) follows at once from (9) and $n_{ks}\geqq 1$.

(d) From inequality (a) and the fact that min $(m_{ks}, n_{ks})\geqq 1$ (for $k=1, 2, 3$ and $s=1, 2, \cdots, r_k$) we conclude

$$r_1+r_2+r_3\leqq m+n. \tag{10}$$

(10) together with $r_k\geqq 2$ from (c) yields (d).

(e) From (b) we conclude

$$m\leqq r_1+r_2+r_3-2. \tag{11}$$

(11) together with $r_k\leqq n$ from (c) yields (e).

(f) Suppose we should have $(m_{k1}, m_{k2}, \cdots, m_{kr_k})=(n_{k1}, n_{k2}, \cdots, n_{kr_k})$ for two values of k, say $k=1$ and $k=2$. We may then assume that $w_1=0$ and $w_2=\infty$. Thus $f(z)$ and $g(z)$ have the same zeros and the same poles, including their multiplicities, and hence $f(z)\equiv cg(z)$ with $c=$const. Since $f(z_{31})=g(z_{31})=w_3$ $(w_3\neq 0, \neq\infty)$ we have $c=1$ which contradicts $f(z)\not\equiv g(z)$. This proves condition (f).

We remark that the inequalities (a) to (e) in the form in which they are written above cannot be improved. Indeed, in Example I below both sides of these inequalities are actually equal. As to condition (f), the relation $(m_{k1}, m_{k2}, \cdots, m_{kr_k})=(n_{k1}, n_{k2}, \cdots, n_{kr_k})$ may actually be satisfied for one k, as shown by Example III.

We shall now give three examples of pairs of rational functions with identical distributions with respect to three values of the dependent variable. We shall briefly describe the procedure for finding such examples. For notations in what follows, we refer to the above table.

We select degrees m and n with $m\geqq n$ and satisfying the conditions of Theorem III. Then we select positive integers r_k, m_{ks} and n_{ks}, com-

patible with (8) and (9) as well as with all the conditions of Theorem III. By the principle of linear substitutions of w and z we may then assign any three arbitrary values to the three w_k, say $w_1=0$, $w_2=\infty$, $w_3=1$, and three arbitrary values to three arbitrarily selected ones among the z_{ks}, say $z_{11}=1$, $z_{21}=\infty$, $z_{22}=0$. (Observe that, by condition (c), $r_2\geqq 2$.) With the other z_{ks} as unknowns and with two unknown constants a and b we have then in view of the distributions with respect to $w_1=0$ and $w_2=\infty$

$$(12)\quad f(z)=[(z-1)^{m_{11}}\prod_{s=2}^{r_1}(z-z_{1s})^{m_{1s}}]/[az^{m_{22}}\prod_{s=3}^{r_2}(z-z_{2s})^{m_{2s}}],$$

$$(13)\quad g(z)=[(z-1)^{n_{11}}\prod_{s=2}^{r_1}(z-z_{1s})^{n_{1s}}]/[bz^{n_{22}}\prod_{s=3}^{r_2}(z-z_{2s})^{n_{2s}}]$$

where the product in the denominator to the right of (12) and (13) is to be unity if $r_2=2$. Now the distribution with respect to $w_3=1$ gives

$$(14)\quad (z-1)^{m_{11}}\prod_{s=2}^{r_1}(z-z_{1s})^{m_{1s}}-az^{m_{22}}\prod_{s=3}^{r_2}(z-z_{2s})^{m_{2s}}=\prod_{s=1}^{r_3}(z-z_{3s})^{m_{3s}},$$

$$(15)\quad (z-1)^{n_{11}}\prod_{s=2}^{r_1}(z-z_{1s})^{n_{1s}}-bz^{n_{22}}\prod_{s=3}^{r_2}(z-z_{2s})^{n_{2s}}=\prod_{s=1}^{r_3}(z-z_{3s})^{n_{3s}}.$$

If now the coefficients of equal powers of z on both sides of (14) and (15) are put equal to one another we obtain $m+n$ equations for the $r_1+r_2+r_3-1$ unknowns z_{ks}, a and b.[6] Each solution of these equations in which the z_{ks} are different from each other and from 0 and 1 and in which a and b are different from 0 corresponds to a pair of functions of the desired type.

By condition (c) of Theorem III, the smallest value for n is 2. If $n=2$ we have, by conditions (d) and (e), $m=4$. The above procedure then leads to the following example:

EXAMPLE I. $f(z)=(z-1)^3(z+3)/16z$, $g(z)=(z-1)(z+3)/4z$.

w_k	z_{ks}	m_{ks}	n_{ks}
0	$1,-3$	3, 1	1, 1
∞	$\infty, 0$	3, 1	1, 1
1	$-1, 3$	3, 1	1, 1

[6] From (10) it follows that we have at least one more equation than unknowns.

We remark without proof that the above pair is essentially the only pair with $m=4$ and $n=2$. That is to say that first no other combinations of m_{ks} and n_{ks} are possible (except permutations of the above) and that secondly all other pairs of degrees $m=4$ and $n=2$ are obtained from the above by a suitable linear substitution of w and z.

The next lowest degrees, according to condition (d), are $m=3, n=3$. These values lead to the following example.

EXAMPLE II.

$$f(z) = (z-1)^2(z+2)/3(\rho^2-1)z, \quad g(z) = z-1)(z+2)^2/3(1-\rho)z^2, \qquad \rho = -\tfrac{1}{2} + \tfrac{1}{2}i3^{1/2}.$$

w_k	z_{ks}	m_{ks}	n_{ks}
0	$1, -2$	2, 1	1, 2
∞	$\infty, 0$	2, 1	1, 2
1	$\rho, -2\rho$	2, 1	1, 2

Example II is again essentially the only pair of functions with $m=3$, $n=3$ in the same sense as Example I above.

The case $m=4$, $n=4$ leads to two interesting examples. One of them is the following.

EXAMPLE III. $f(z)=(z-1)^3(z+1)/4z$, $g(z)=(z-1)(z+1)^3/4z^3$.

w_k	z_{ks}	m_{ks}	n_{ks}
0	$1, -1$	3, 1	1, 3
∞	$\infty, 0$	3, 1	1, 3
1	$i, -i, 1+2^{1/2}, 1-2^{1/2}$	1, 1, 1, 1	1, 1, 1, 1

In this example the relation of condition (f) of Theorem III is actually satisfied for one k, namely, $k=3$.

Another example with $m=4$, $n=4$ can be obtained from Example I by replacing z by $-3/z$ in the function $f(z)$. It is easily seen that $f(-3/z)$ and $f(z)$ form a pair of functions of the desired type. Thus the three functions[7] $f(z)$, $f(-3/z)$ and $g(z)$ of Example I form a *triplet* of rational functions with identical distributions with respect to three values of the dependent variable.

[7] Note that $g(-3/z)\equiv g(z)$.

The last mentioned example raises the question as to whether there could be an infinite number of rational functions which have given distributions with respect to three given values. This question is answered as follows.

THEOREM IV. *There can be only a finite number of rational functions which have given distributions with respect to three given values of the dependent variable.*

PROOF. Let $z_{k\nu}$ be the given distributions with respect to the given values w_k (see table preceding Theorem III). Let $w=f(z)$ be any rational function having these distributions, let m be its degree and $m_{k\nu}$ be its multiplicity at $z=z_{k\nu}$. Then, by the proof of condition (b) in Theorem III, m cannot exceed $r_1+r_2+r_3-2$. This leaves only a finite number of values for m, hence by (8) there is only a finite number of possible combinations of $m_{k\nu}$. Each such combination can, according to condition (f) of Theorem III, lead to at most one function $f(z)$. This proves the theorem.

It may be noted in conclusion that any pair of functions $w=f(z)$ and $w=g(z)$ of the above-mentioned type leads to infinitely many more pairs of the same kind, other than those obtained by linear substitutions of z and w. For, if $\phi(z)$ is any rational function whatever, then $w=f(\phi(z))$ and $w=g(\phi(z))$ have again identical distributions with respect to the same three values of w as the original functions $w=f(z)$ and $w=g(z)$. This shows, in particular, that the degrees m and n of such pairs of functions may exceed any given number.

CORNELL UNIVERSITY

CONTINUA OF FINITE DEGREE AND CERTAIN PRODUCT SETS[1]

O. G. HARROLD, JR.[2]

The continua of finite degree have been studied and identified with certain well known classes of continua in a paper by G. T. Whyburn.[3] The author has shown that the continua of finite degree are identical with the continua homeomorphic to a continuum of finite length.[4] The object of the present note is to obtain other internal characterizations of these continua.

The symbol M will represent a (compact) continuum. The continuum M is said to be of finite degree at the point $p \in M$ provided that to each $\epsilon > 0$ there corresponds an *uncountable* family of neighborhoods (U) of p such that (a) the diameter of each neighborhood is less than ϵ, (b) each $F(U)$ is finite, where $F(U)$ is the boundary of U, and (c) for any pair of neighborhoods U and V either $\overline{U} \subset V$ or $\overline{V} \subset U$. If every point is of finite degree, the continuum M is said to be of finite degree. The characterization which we find most useful below is that a continuum M is of finite degree if and only if every subcontinuum contains uncountably many local separating points of M.

It will be shown that the classes of continua defined by each of the following properties are identical with the continua of finite degree.

PROPERTY N^0. *M is locally connected and to each pair of closed, disjoint subsets A and B in M there corresponds a finite collection of disjoint, perfect sets $H^1, H^2, \cdots, H^k$ such that any continuum K in M intersecting both A and B contains some H^i.*

PROPERTY Q. *If K and K_i, $(i = 1, 2, \cdots)$, are nondegenerate continua in M with $\lim K_i = K$, there exists an integer n such that $\prod_n^\infty K_i$ is an uncountable set.*

It will be noted that the Property N^0 is highly analogous to Property N which characterizes the locally connected continua such that no true cyclic element has a continuum of condensation.[5]

[1] Presented to the Society, December 29, 1939.

[2] National Research Fellow.

[3] G. T. Whyburn, *Concerning continua of finite degree and local separating points*, American Journal of Mathematics, vol. 57, pp. 11–16.

[4] See abstract 45-9-321, this Bulletin.

[5] This concept is due to R. L. Moore. See his *Fundamental Point Set Theorems*, Rice Institute Pamphlets, vol. 23, no. 1, 1936.

For this characterization see G. T. Whyburn, *On continua of condensation*, American Journal of Mathematics, vol. 58, pp. 705–708.

THEOREM A. *A continuum is of finite degree if and only if it has Property* N^0.

PROOF. A continuum of finite degree has the property that any pair of closed, disjoint subsets A and B in M can be separated by a finite number of points $q^1, q^2, \cdots, q^n$ which are local separating points of degree two.[3] Suppose M is of finite degree. Set $d=\min \rho(q^i, q^j)$, $i\neq j$, $\rho(q^i, A+B)$. Since q^i is of degree two, there exists an uncountable family of neighborhoods $[U(q^i)]$ such that (a) $\delta(U)<\frac{1}{2}d$, (b) $F(U)$ consists of at most two points, and (c) for any pair of distinct elements U and V of the family either $\overline{U}\subset V$ or $\overline{V}\subset U$. Set

$$Y^i=\overline{\sum F(U(q^i))},$$

i fixed. Let H^i be a perfect subset of Y^i. There is thus determined a finite collection of disjoint, perfect sets $H^1, H^2, \cdots, H^n$. Let K be any continuum in M intersecting both A and B. Since M is of finite degree (thus hereditarily locally connected), K may be taken as an arc. Since K contains some q^i, it contains the corresponding H^i.

Conversely, suppose M has Property N^0. To prove M is of finite degree it suffices to show that every nondegenerate subcontinuum in M contains uncountably many local separating points of M. Let K be a continuum in M containing the distinct points x and y. Set $A=x$, $B=y$. Then by Property N^0 there exists disjoint, perfect sets $H^1, H^2, \cdots, H^n$ such that $K\supset H^1$ (say). If $K\cdot L$ is uncountable, where L is the set of local separating points of M, our end is attained. If $K\cdot L$ is countable, there is a point $z^1\in H^1(M-L)$. Set $d=1/2 \min \rho(x, H^i), \rho(y, H^i), \rho(H^i, H^j), i\neq j$. Let V be a region in M containing z^1 which is of diameter less than d. Set $d^1=\rho[z^1, F(V)]$. Let W be the $\frac{1}{2}d^1$ neighborhood of z^1, that is, the set of all points b such that $\rho(z^1, b)<\frac{1}{2}d^1$. Let X^1 and Y^1 be the components of $K-KW$ containing x and y, respectively. Since X^1 and Y^1 each contain a point of V and $z^1\in M-L$, there exists an arc s^1 in $V-z^1$ joining X^1 and Y^1. Hence $K^1=X^1+s^1+Y^1$ is a continuum from x to y not containing H^1. If K^1 contains no other H^i, we have a contradiction to Property N^0. If $K^1\supset H^2$, then $H^2\subset X^1+Y^1$ on account of the manner of selection of s^1. Since $K\cdot L$ is countable, $(X^1+Y^1)L$ is countable, hence there exists a point $z^2\in H^2(M-L)$. Using the same d as before, a continuum K^2 is constructed (with K^1 replacing K) such that K^2 contains neither H^1 nor H^2 and K^2 intersects both A and B. Performing these steps (with z^2 replacing z^1) will clearly give a continuum $K^2=X^2+s^2+Y^2$ not containing H^2, and since $s^2H^1=0$ and $X^2+Y^2\subset K^1$, we have K^2 contains neither H^1 nor H^2. After a finite number of such steps a con-

tinuum K^i is obtained which intersects both A and B and contains no H^i. This completes the proof that having Property N^0 is equivalent to being of finite degree.

THEOREM B. *A continuum is of finite degree if and only if it has Property* Q.

PROOF. Let M be of finite degree. Since this implies that M is hereditarily locally connected, it may be assumed without loss of generality that the continua K_i such that $\lim K_i = K$ are arcs. Let the end-points of K_i be a_i and b_i. Suppose $\lim a_i = x$ and $\lim b_i = y$. Since K by assumption is a nondegenerate continuum, we may take $x \neq y$. In a continuum of finite degree the points of degree two are dense on every subcontinuum, hence let z be a point of degree two in $K-(x+y)$. Let X, Y and Z be neighborhoods of x, y and z respectively, such that $(\overline{X}\cdot\overline{Z}+\overline{X}\cdot\overline{Y}+\overline{Y}\cdot\overline{Z})=0$. To Z there corresponds an uncountable family of neighborhoods (U) such that $\overline{U}\subset Z$ and $F(U)$ consists of at most two points. There is a definite $V=V(z)$ such that $\overline{V}\subset U$ for uncountably many U. Since $z\in\lim K_i$, there is an integer n such that arcs K_i, $i\geqq n$, intersect V, hence if we take $m\geqq n$ so large that $a_i\in X$, $b_i\in Y$ for $i\geqq m$, then K_i must contain two boundary points of each U for which $V\subset U\subset Z$. Thus $\prod_m^\infty K_i$ is uncountable.

To show that Property Q implies finite degree we show that infinite degree implies Property non-Q. It is clear that a continuum containing a convergence continuum has Property non-Q, hence we need consider only hereditarily locally connected continua. Let T be an arc in the continuum M which contains only a countable number of the local separating points of M. For each positive integer n let W_n be the set of all points of M at a distance not greater than $1/n$ from T. Let M_n be the component of W_n which contains T. Let L_n be the set of all points which separate the end-points a and b of T in M_n. Then in M_n there exist arcs s_n and t_n from a to b such that[6] $s_nt_n=a+L_n+b$. Set $K_{2n-1}=s_n$ and $K_{2n}=t_n$. Then $\lim K_n=T$. But $\prod_n^\infty K_i$ is at most countable, no matter what positive integer n is. For K_{2n-1} and K_{2n} have only a countable number of common points, namely, $a+L_n+b$. Hence M does not have Property Q.

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[6] G. T. Whyburn, *Some properties of continuous curves*, this Bulletin, vol. 33 (1927), pp. 305–308. See Theorem 3.

NOTE ON SOME ELEMENTARY PROPERTIES OF POLYNOMIALS

P. ERDÖS

In a previous paper T. Grünwald[1] and I proved that if $f(x)$ is a polynomial of degree $n \geqq 2$ and satisfies the following conditions:

$$\text{(1)} \qquad \begin{aligned} &\text{all roots of } f(x) \text{ are real, } \quad f(-1) = f(+1) = 0, \\ &f(x) \neq 0 \quad \text{for} \quad -1 < x < 1, \quad \max_{-1 \leq x \leq 1} f(x) = 1, \end{aligned}$$

then

$$\text{(2)} \qquad \int_{-1}^{+1} f(x) \leqq \frac{4}{3}.$$

Equality occurs only for $f(x) = 1 - x^2$.

This result can be generalized as follows: Suppose $f(x)$ satisfies (1) and let $f(a) = f(b) = d \leqq 1$, $-1 < a < b < 1$; then

$$\text{(3)} \qquad b - a \leqq 2(1 - d)^{1/2}.$$

Again equality occurs only for $f(x) = 1 - x^2$. It is clear that (2) follows from (3) by integration with respect to d.

PROOF. Instead of (3) we prove the following slightly more general result: Let $f(x)$ satisfy (1), and determine the greatest positive constant c_f such that

$$f(a)f(a + c_f) = d^2, \quad -1 < a < a + c_f < 1;$$

then

$$\text{(4)} \qquad c_f \leqq 2(1 - d)^{1/2}.$$

Equality holds only for $f(x) = 1 - x^2$, $a = -(1-d)^{1/2}$.

Suppose there exists a polynomial of degree $n > 2$ satisfying (1) with $c_f \geqq 2(1-d)^{1/2}$; then we will prove that there exists a polynomial of degree $n-1$ with $c_f > 2(1-d)^{1/2}$; and this proves (4) since it is easy to prove that (4) is satisfied for polynomials of second degree, that is, for $1 - x^2$.

Denote the roots of $f(x)$ by $x_1 = -1, x_2 = 1, x_3, \cdots, x_n$ and suppose first that for $i > 2$ the x_i are not all of the same sign. Let x_n be the largest positive root and x_{n-1} the smallest negative root, and denote by y the root of $f'(x)$ in $(-1, +1)$. Consider the polynomial of degree n

[1] Annals of Mathematics, (2), vol. 40 (1939), pp. 537–548.

$$\phi(x) = c\frac{f(x)(x-y)^2}{(x-x_n)(x-x_{n-1})},$$

where we choose c so that $\phi(x) \geqq 0$ for $-1 \leqq x \leqq 1$. Then it is easy to see that for large x, $\phi(x)$ and $f(x)$ have opposite signs. Thus their leading coefficients have opposite signs. Hence it is possible to choose c such that the polynomial $F(x) = f(x) + \phi(x)$ is of degree $n-1$. Since $n-2$ of its roots are real it can have only real roots, and since $F'(y) = 0$, $F(y) = 1$, it follows that $\max_{-1 \leqq x \leqq 1} F(x) = 1$. Thus $F(x)$ satisfies (1) (obviously $F(x) \neq 0$ for $-1 < x < 1$) and $F(x) \geqq f(x)$ in -1, $+1$, equality occurring only for -1, y, $+1$. Thus $c_F > c_f$. Hence we may suppose that for $i > 2$ all the x_i are of the same sign; without loss of generality we may suppose them negative. Suppose that

$$f(a)f(b) = d^2, \qquad b - a = c_f.$$

We can suppose that $-1 < a < y < b < 1$. We now prove that

$$b - y < y - a. \tag{5}$$

For if not then

$$|f'(b)| > |f'(a)|, \qquad f(b) < f(a), \tag{6}$$

that is,

$$|f'(b)| = \left|(b-y)\prod_{i=1}^{n-2}(b-y_i)\right|, \qquad |f'(a)| = \left|(y-a)\prod_{i=1}^{n-2}(y_i-a)\right|,$$

$$y > y_i, \quad i = 1, 2, \cdots, n-2,$$

where $b - y \geqq y - a$ and all other factors in $|f'(b)|$ are greater then the corresponding factors in $f'(a)$. This proves the first inequality of (6). To prove the second inequality we remark that from what has just been said it follows that for $u_1 - y = y - u_2$, $-1 < u_2 < y < u_1 < 1$, we have

$$|f'(u_1)| > |f'(u_2)|,$$

and since $b - z \geqq y - a$ the second inequality follows by integration.

By simple calculation it follows from (6) that

$$f(b-\epsilon)f(a-\epsilon) > f(a)f(b) = d^2, \qquad \epsilon > 0 \text{ sufficiently small.}$$

Thus $b - a < c_f$. This contradiction proves (5).

Let x_n be the root of $f(x)$ with greatest absolute value. Consider the new polynomial

$$f_1(x) = c\frac{x - x_n'}{x - x_n}f(x), \qquad x_n' = x_n - \delta, \delta > 0,$$

where c is chosen in such a way that $\max_{-1\le x\le 1} f_1(x)=1$. Then we prove

$$c_{f_1} > c_f. \tag{7}$$

To show (7) it will suffice to show that c_f is an increasing function of $|x_n|$. Choose δ so small that if we denote by $y^{(1)}$ the root of $f_1'(x)$ in $(-1, +1)$ we have $b-y_1<y_1-a$ (it is clear that $y_1<y$).

Put now

$$c = \left| \frac{1}{f(y_1)} \frac{x_n - y_1}{x_n - \delta - y_1} \right|.$$

(Evidently $cf_1(x)$ satisfies (1).)

Now

$$c^2 f_1(a) f_1(b) = c^2 \frac{a - x_n + \delta}{a - x_n} \frac{b - x_n + \delta}{b - x_n} f(a) f(b)$$

$$> \left(1 + \frac{\delta}{a - x_n}\right)\left(1 + \frac{\delta}{b - x_n}\right)\left(\frac{1}{1 + \delta/y_1 - x_n}\right)^2 f(a) f(b)$$

(that is, $f(y_1)<1$). But from (5) we have

$$\delta\left(\frac{1}{a - x_n} + \frac{1}{b - x_n}\right) > \frac{2\delta}{\left(\frac{a+b}{2} - x_n\right)} > \frac{2\delta}{y_1 - x_n}$$

and

$$\frac{\delta^2}{(a - x_n)(b - x_n)} > \frac{\delta^2}{\left(\frac{a+b}{2} - x_n\right)^2} > \frac{\delta^2}{(y_1 - x_n)^2}.$$

Thus

$$c^2 f_1(a) f_1(b) > f(a) f(b) = d^2.$$

Hence (7) is proved.

If $|x_n|$ tends to infinity $f(x)$ tends to $F(x)=f(x)/(x-x_n)$, which is of degree $n-1$. From (7) it follows that $c_F > c_f$, which proves the theorem.

Let $f(x)$ be a polynomial of degree n all the roots of which are in the interval $(-1, +1)$; and further let $\max_{-1\le x\le 1} |f(x)| = 1$. For which polynomial is

$$\int_{-1}^{+1} |f(x)|$$

maximal? I was not able to answer this question but it seems very likely that the maximum is reached for $f(x)=T_n(x/c)$, where $c=1/x_n$, and x_n is the greatest root of $T_n(x)$ (the nth Tchebicheff polynomial). Hence $T_n(1/c)=T_n(-1/c)=0$ and all other roots of $T_n(x/c)$ are in $(-1, +1)$. It is easy to see that $T_n(x)$ satisfies the following condition: Let x_i and x_{i+1} be two consecutive roots of $T_n(x)$; then

$$\frac{1}{x_{i+1}-x_i}\int_{x_i}^{x_{i+1}} |T_n(x)| = d_n,$$

where d_n is independent of i, and $\lim d_n=2/\pi$.

This fact suggests the following conjecture which is a generalization of the previous one: Let $f(x)$ be a polynomial of degree n all the roots of which are in $(-1, +1)$, such that $\max_{-1\leq x\leq 1}|f(x)|=1$ and let x_i and x_{i+1} be two consecutive roots of $f(x)$; then

$$\int_{x_i}^{x_{i+1}} |f(x)| \leqq d_n(x_{i+1}-x_i).$$

Equality holds only for $T_n(cx)$.

It seems very likely that the following result holds: Let $\phi(\theta)$ be a trigonometric polynomial all the roots of which are real, further let $\max_{0\leq\theta\leq 2\pi}|\phi(\theta)|=1$. Then

$$\int_0^{2\pi} |\phi(\theta)| \leqq 4.$$

Let $f(x)$ be a polynomial of degree n with leading coefficient 1 and all roots in $(-1, +1)$; then the sum of the intervals in $(-1, +1)$ for which $|f(x)|\geqq 1$ does not exceed 1. The proof is quite simple. Evidently

$$f(x)f(-x)=\prod_{i=1}^{n}(x_i^2-x^2)\leqq 1 \qquad \text{for } |x|\leqq 1,$$

equality occurring only for $x=0$, $|x_i|=1$. Thus one of the numbers $f(x)$ or $f(-x)$ is less than 1, which establishes the result. It is also easy to see that if the sum of the intervals in question is exactly 1 then $f(x)=(1\pm x)^n$. It would not be difficult to prove the following slightly more general result: Let $f(x)$ have leading coefficient 1 and all roots in $(-1, +1)$; then if $-1<a<0<b<1$ at least one of the numbers $|f(a)|$ or $|f(b)|$ is less than 1. These problems become very much more difficult if instead of the interval $-1, +1$ we consider the unit circle. The question would be to determine the polynomial (or poly-

nomials) of degree not greater than n with leading coefficient 1 and all roots in the unit circle such that the area of the set of points for which $|f(x)| \geqq 1$ shall be as big as possible. A first guess would be $f(x)=(x-a)^n$, $|a|=1$, but it can be shown that for sufficiently large n this is not the case. The complete solution of this problem seems difficult.

Mr. Eröd[2] proved that there exists a constant c independent of n such that for a polynomial of degree n satisfying the above conditions the area of the set of points for which $|f(x)| \leqq 1$ is not less than c. The best value of c is not known.

INSTITUTE FOR ADVANCED STUDY

[2] Oral communication.

ON UNCONDITIONAL CONVERGENCE IN NORMED VECTOR SPACES

T. H. HILDEBRANDT

Suppose X is a complete normed vector or Banach space of elements x. Orlicz[1] has given the following two definitions of unconditional convergence of an infinite series $\sum_n x_n$ of elements from X and proved their equivalence:

A. $\sum_n x_n$ is unconditionally convergent if and only if any rearrangement of the series is convergent.

B. $\sum_n x_n$ is unconditionally convergent if and only if $\sum_k x_{n_k}$ converges, where $\{x_{n_k}\}$ is any subsequence of $\{x_n\}$.

Pettis[2] has shown that either of these conditions is equivalent to the statement:

C. Every subseries of $\sum_n x_n$ is weakly convergent to an element of X, that is, $\{x_{n_k}\}$ implies the existence of an element x_σ such that, for every $\bar{x}$ of the conjugate space $\overline{X}$, $\sum_k \bar{x}(x_{n_k}) = \bar{x}(x_\sigma)$.

In proving this equivalence, he shows that condition C implies the following:

D. $\lim_n \sum_{m=n}^{\infty} \bar{x}(x_m) = 0$ uniformly for $\|\bar{x}\| = 1$.

E. H. Moore[3] has shown that for real, complex or quaternionic numbers, absolute and therefore unconditional convergence is equivalent to the following definition of convergence:

Let σ be any finite subset $n_1, \cdots, n_k$ of the positive integers, and denote $\sum_{i=1}^{k} x_{n_i}$ by $\sum_\sigma x_n$. Then

E. $\sum x_n$ converges in the σ-sense, if $\lim_\sigma \sum_\sigma x_n$ exists, where the limit is the Moore-Smith limit, and $\sigma_1 \geqq \sigma_2$ means that σ_1 contains all of the numbers in σ_2.[4]

Obviously the Moore-Smith limit can be extended to normed vector spaces, and the fundamental properties carry over. It is the purpose of this note to show that convergence in the σ-sense is equivalent to each of the conditions A, B, and D, that is, A, B, D and E are equivalent definitions of unconditional convergence, to which the

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[1] *Ueber unbedingte Konvergenz in Funktionenräumen*, Studia Mathematica, vol. 4 (1933), pp. 33–38.

[2] *Integration in vector spaces*, Transactions of this Society, vol. 44 (1938), pp. 281–282.

[3] *General Analysis*, Memoirs of the American Philosophical Society, vol. 1, part 2, 1939, p. 63.

[4] See Alaoglu, Annals of Mathematics, (2), vol. 41 (1940), p. 259, where a similar definition for weak unconditional convergence is given.

Pettis result adds C as a fifth equivalent definition. We consider definition E the most elegant of the definitions of unconditional convergence.[5]

A *is equivalent to* E. Assume that every rearrangement of $\sum x_n$ is convergent and suppose if possible $\lim_\sigma \sum_\sigma x_n$ is not equal to $x=\sum_n x_n$, summed in its natural order. Then there exists an $e>0$, such that for every σ there exists a $\sigma' \geqq \sigma$ such that $\|x-\sum_{\sigma'} x_n\| \geqq e$. Let n_0 be chosen so that for $n \geqq n_0$ we have $\|x-\sum_1^n x_m\| < e/2$. Let σ_1 be the set $1, 2, \cdots, n_0$, and σ_1' chosen so that $\|x-\sum_{\sigma_1'} x_n\| \geqq e$. Let σ_2 include all of the integers less than or equal to any integer in σ_1'. Repeating the process produces a σ_1', a σ_3, and so on, and defines a rearrangement of $\sum x_n$, namely, $\sigma_1, \sigma_1'-\sigma_1, \sigma_2-\sigma_1', \cdots$ (where $\sigma-\sigma'$ means the elements of σ not in σ'), which is not a convergent series since

$$\left\| \sum_{\sigma_m} x_n - \sum_{\sigma_{m'}} x_n \right\| = \left\| \sum_{\sigma_{m'}-\sigma_m} x_n \right\| \geqq e/2.$$

Since $\lim_\sigma \sum_\sigma x_n$ is unique when it exists, it follows at once that every rearrangement of $\sum_n x_n$ converges to the same limit.[6]

Conversely, suppose $\sum_\sigma x_n$ approaches a limit x, that is, for every $e>0$, there exists a σ_e such that if $\sigma \geqq \sigma_e$ then $\|\sum_\sigma x_n - x\| \leqq e$. Let $\{x_{n_k}\}$ be any rearrangement of $\{x_n\}$. Then we need only to choose k_e so that the set of integers $n_1, \cdots, n_{k_e}$ includes all of the integers in σ_e to be assured that for $k' \geqq k_e$ it is true that

$$\left\| \sum_{k=1}^{k'} x_{n_k} - x \right\| \leqq e.$$

B *is equivalent to* E. Suppose every subseries $\sum_k x_{n_k}$ converges but $\lim_\sigma \sum_\sigma x_n$ does not exist, that is, there exists an $e>0$ such that for every σ there exist $\sigma', \sigma'' \geqq \sigma$ such that $\|\sum_{\sigma'} x_n - \sum_{\sigma''} x_n\| > e$. If $\sigma'+\sigma''$ denotes the set including all elements of σ' and σ'', then either $\|\sum_{\sigma'} x_n - \sum_{\sigma'+\sigma''} x_n\| \geqq e/2$ or $\|\sum_{\sigma''} x_n - \sum_{\sigma'+\sigma''} x_n\| \geqq e/2$, that is, we can assume $\sigma' \geqq \sigma'' \geqq \sigma$. We obtain a nonconvergent subseries as follows: Take $\sigma_1=1$. This gives rise to $\sigma_1' \geqq \sigma_1'' \geqq \sigma_1$ so that $\|\sum_{\sigma_1'} x_n - \sum_{\sigma_1''} x_n\| > e$. Let $n_1, \cdots, n_{k_1}$ be the elements of $\sigma_1' - \sigma_1''$. Take $\sigma_2=\sigma_1'$, and $n_{k_1+1}, \cdots, n_{k_2}$ to be the elements of $\sigma_2' - \sigma_2''$. Proceeding in this manner we get a series $x_{n_1}+x_{n_2}+\cdots+x_{n_{k_1}}+\cdots+x_{n_{k_2}}+\cdots$ such that $\|\sum_{k_m+1}^{k_{m+1}} x_{n_k}\| > e$, that is, one which is not convergent.

[5] In view of the results of Orlicz and Pettis, it would be sufficient to prove that D implies E implies A. For the sake of completeness and elegance we have preferred to prove each equivalence separately.

[6] See Orlicz, Studia Mathematica, vol. 1 (1929), p. 242.

Conversely if $\lim_\sigma \sum_\sigma x_n$ exists and $\{x_{n_k}\}$ is any subsequence of x_n, then if k_ϵ is chosen so that n_{k_ϵ} is larger than any integer appearing in the σ_ϵ involved in the definition of the limit, it will be certain that for $k' \geqq k_\epsilon$ and any l

$$\left\| \sum_{k=k'}^{k'+l} x_{n_k} \right\| \leqq 2\epsilon,$$

that is, $\sum_k x_{n_k}$ is convergent.

D *is equivalent to* E. Suppose $\lim_n \sum_{m=n}^{\infty} |\bar{x}(x_m)| = 0$ uniformly for $\|\bar{x}\| = 1$. We demonstrate that $\lim_{\sigma_1,\sigma_2} \|\sum_{\sigma_1} x_n - \sum_{\sigma_2} x_n\| = 0$. This is obviously equivalent to showing that $\lim_{\sigma_1 \geqq \sigma_2} \|\sum_{\sigma_1} x_n - \sum_{\sigma_2} x_n\| = 0$, since

$$\left\| \sum_{\sigma_1} x_n - \sum_{\sigma_2} x_n \right\| \leqq \left\| \sum_{\sigma_1} x_n - \sum_{\sigma_1+\sigma_2} x_n \right\| + \left\| \sum_{\sigma_2} x_n - \sum_{\sigma_1+\sigma_2} x_n \right\|.$$

Let n_ϵ be such that for $n \geqq n_\epsilon$ we have $\sum_n^{\infty} |\bar{x}(x_m)| \leqq \epsilon$ if $\|\bar{x}\| = 1$, and take $\sigma_\epsilon = 1, 2, \cdots, n_\epsilon$ with $\sigma_1 \geqq \sigma_2 \geqq \sigma_\epsilon$. Then $\sum_{\sigma_1-\sigma_2} |\bar{x}(x_n)| \leqq \epsilon$ and so $|\bar{x}(\sum_{\sigma_1-\sigma_2} x_n)| \leqq \epsilon$. Since this is uniform for $\|\bar{x}\| = 1$, it follows that

$$\left\| \sum_{\sigma_1-\sigma_2} x_n \right\| = \text{l.u.b.} \left[\left| \bar{x}\left(\sum_{\sigma_1-\sigma_2} x_n \right) \right| \text{ for } \|\bar{x}\| = 1 \right] \leqq \epsilon.$$

Conversely suppose $\lim_\sigma \sum_\sigma x_n$ exists. Then for $\sigma_1 \geqq \sigma_2 \geqq \sigma_\epsilon$ we have $\|\sum_{\sigma_1-\sigma_2} x_n\| \leqq \epsilon$. Take n_ϵ greater than the largest integer in σ_ϵ. Let $m \geqq n_\epsilon$ and $\|\bar{x}\| = 1$. Denote by $\sigma_{1\epsilon}$ those of the integers $m, m+1, \cdots, m+k$, for which $\bar{x}(x_n) \geqq 0$, and by $\sigma_{2\epsilon}$ the integers for which $\bar{x}(x_n) < 0$. Then

$$\sum_m^{m+k} |\bar{x}(x_n)| = \sum_{\sigma_{1\epsilon}} |\bar{x}(x_n)| + \sum_{\sigma_{2\epsilon}} |\bar{x}(x_n)|$$

$$= \left| \bar{x}\left(\sum_{\sigma_{1\epsilon}} (x_n) \right) \right| + \left| \bar{x}\left(\sum_{\sigma_{2\epsilon}} x_n \right) \right| \leqq \left\| \sum_{\sigma_{1\epsilon}} x_n \right\| + \left\| \sum_{\sigma_{2\epsilon}} x_n \right\| \leqq 2\epsilon;$$

that is, $\lim_n \sum_n^{\infty} |\bar{x}(x_n)| = 0$, uniformly for $\|\bar{x}\| = 1$. A similar procedure would take care of the case in which $\bar{x}(x)$ were complex valued.

We note that condition D is equivalent to the statement that the linear operations (functionals) $\bar{x}$ for which $\|\bar{x}\| = 1$ map the sequence x_n on a compact subset of l, the space of absolutely convergent series, that is, any unconditionally convergent series can be interpreted as a completely continuous transformation on the adjoint space $\overline{X}$ to the space l.[7]

[7] See Dunford, Transactions of this Society, vol. 44 (1938), p. 322.

Further it can be deduced from condition B that if $x(\sigma)=\sum_k x_{n_k}$ where $\sigma=n_1, n_2, \cdots$ then $x(\sigma)$ is a completely additive set function on subsets of the integers to the space X.

E. H. Moore, in the reference mentioned above, considers the case in which the set of integers $1, 2, \cdots, n, \cdots$ is replaced by a general set: $\mathfrak{P}$, and then defines a general sum $\sum x(p)$ by an obvious generalization. He shows that if $\sum x(p)$ exists, then $x(p)$ is zero except at a denumerable set of elements $p_1, \cdots, p_n, \cdots$ and $\sum_n |x(p_n)|$ exists. This result is extensible to the case where x is on $\mathfrak{P}$ to a linear normed complete space in the form:

If x is on $\mathfrak{P}$ to X and if $\sum x(p)$ exists in the sense that $\lim_\sigma \sum_\sigma x(p)$ exists, where the σ are finite subsets of $\mathfrak{P}$, then $x(p)$ differs from zero at most at a denumerable set of elements $p_1, \cdots, p_n, \cdots$ and $\sum x(p_n)$ is unconditionally convergent.

The first part of this theorem depends on the fact, easily derived by a slight change in the proof of "E implies D" above, that if $\sum_p x(p)$ exists then $\lim_\sigma \sum_\sigma |\bar{x}(x_p)|$ exists uniformly for $\|\bar{x}\|=1$, that is, for every $e>0$ there exists a σ_e such that if $\sigma_1 \geqq \sigma_2 \geqq \sigma_e$ and $\|\bar{x}\|=1$ then $\sum_{\sigma_1-\sigma_2} |\bar{x}(x_p)| \leqq e$. Let $\mathfrak{P}_0$ be the sum of the sets σ_e for $e=1/n$. This set will be denumerable. If p is not of $\mathfrak{P}_0$ then $|\bar{x}(x_p)| \leqq 1/n$ for all n and $\|\bar{x}\|=1$, that is, $x_p=0$. The second part of the theorem is obvious.

UNIVERSITY OF MICHIGAN

A NOTE ON THE LIMIT OF ORBITS[1]

G. E. SCHWEIGERT

Theorem A, stated below, is established in a recent paper[2] by D. W. Hall and the author. Various factors favoring a change in the point of view have enabled the writer to see a simpler proposition which contains Theorem A as a corollary. The purpose of this note is to prove this newer theorem and to point to its implications in the particular case shown by the example at the end.

A single valued continuous transformation $T(M)=M$ of a compact metric space onto itself is said to be *pointwise periodic* provided that for each point x in M there exists a positive integer n such that $T^n(x)=x$. Such a transformation must be one-to-one, hence a homeomorphism. The least positive integer n such that $T^n(x)=x$ is said to be the period of a point x in M and is denoted by $p(T, x)$. By the orbit of a point under T, or more briefly by a *point-orbit*, we shall mean the finite set consisting of a point x of M and all the images $T(x)$, $T^2(x)\equiv TT(x)$, and so on, under T. Similarly if X is a component of an invariant set L, that is, a set L for which $T(L)=L$, then the finite sum of disjoint components $X+T(X)+T^2(X)+\cdots+T^k(X)$ is said to be the *component-orbit* of X relative to L under T. This extention of the notion of a point-orbit[3] is possible since the property of being a component (maximal connected set) in L is invariant under T. In all that follows the brief term component-orbit will be used instead of the original precise form in which an invariant set is chosen first. Furthermore if G is such a component-orbit sets X or K mentioned in close proximity to G may be understood without confusion to be components such as X in the definition above. In other words it is best to think of each G as a function of X or K as if it were written $G(X)$ or $G(K)$, so that these sets may be used freely without accounting for their presence each time. The lower case letters such as x and p will denote points, and point-orbits will bear a subscript, for example, G_x.

These two types of orbit are obviously related in such a fashion that: (a) The number of components in a n orbit G equals at most

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[1] Presented in part to the Society, April 15, 1939.

[2] *Properties of invariant sets under pointwise periodic homeomorphisms*, Duke Mathematical Journal, vol. 4 (1938), pp. 719–724.

[3] See L. Whyburn, *Rotation groups about a set of fixed points*, Fundamenta Mathematicae, vol. 28 (1937), p. 124. This is perhaps the first and certainly a most consistent and fruitful use of this extended notion.

the minimum value of $p(T, x)$ for $x \in X$; (b) If G has k components $T^i(X)$ and if G_x is a point-orbit associated with $x \in X$, then the number of points in $G_x \cdot T^i(X)$ is the same for all values of i in the range 0 to $k-1$. Furthermore the set $G_x \cdot T^i(X)$ is a point-orbit under T^k for each fixed i. It is to be understood that $T^0(X) \equiv X$ as in case for $i=0$ above.

THEOREM A. *If M is a compact metric space, $T(M)=M$ a pointwise periodic transformation, $\{G_{x_k}\}$ a convergent sequence of point-orbits under T with limit set L, and if there is in L a connected set B such that $T(B)=B$, then L is connected.*

The following lemma is stated without proof since it is practically the same as Theorem A and follows from the exact argument used in the proof as published.

LEMMA 1. *If M is a compact metric space, $T(M)=M$ a pointwise periodic transformation, $\{G_{x_k}\}$ a convergent sequence of point-orbits under T with limit set L, and if G_x denotes any point-orbit in L, then L has the property that for every separation*[4] *$L=L^1+L^2$ the sets $G_x \cdot L^1$ and $G_x \cdot L^2$ are non-vacuous.*

We are now in a position to prove an extended form of this result.

LEMMA 2. *If M is a compact metric space, $T(M)=M$ a pointwise periodic transformation, $\{G_{x_k}\}$ a convergent sequence of point-orbits under T with limit set L, and if X is a component of L, then $L=G$ where G is the component-orbit of X under T.*

PROOF. *Case* (i): *There exists an isolated component K in L*, that is, K is a component which is both open and closed in L.

Let G denote the component-orbit of K and consider the equations

$$L = G + (L - G), \qquad G = K + T(K) + \cdots + T^m(K),$$

m a positive integer.

Now T is a homeomorphism on L, and K being isolated in L implies that $T(K)$ and $T^2(K)$, and so on, are also isolated in L so that G is both open and closed in L. This shows that the set $L-G$ must be vacuous, for otherwise $L=G+(L-G)$ is a separation of L contrary to Lemma 1. If $L-G$ is vacuous then $L=G$ and Lemma 2 is true.

Case (ii): *L contains no isolated components.*

We will show that this case is impossible in view of Lemma 1. Let

[4] We say that $M=R+S$ is a separation of M provided neither R nor S is vacuous and $R\bar{S}=0=\bar{R}S$, where $\bar{N}$ is the closure of a set N.

X be any component of L and let m be a positive integer chosen so that $X, T(X), \cdots, T^m(X)$ represent the $(m+1)$ distinct components of the component-orbit G. Now L is not connected, for otherwise L is itself an isolated component in L, and we may therefore choose a separation of L; say $L=L(1)+L(2)$. Neither $L(1)$ nor $L(2)$ is an isolated component in L and hence we may choose separations $L(1)=L(1,1)+L(1,2)$ and $L(2)=L(2,1)+L(2,2)$ so that $L=L(1,1)+L(1,2)+L(2,1)+L(2,2)$. Each of the summands $L(i_1, i_2)$ when $i_1=1, 2$ and $i_2=1, 2$ is an isolated set in L, hence fails to be connected, and the process continues. In general

$$L = \sum_{i_1, i_2, \cdots, i_j} L(i_1, i_2, \cdots, i_j), \qquad \text{where } i_k = 1, 2,$$

while k has the range $1, 2, \cdots, j$; thus L is expressed as the sum of 2^j disjoint sets each isolated in L. Let a fixed value of $j=j^*$ be chosen so that 2^{j^*} is greater than $m+1$. Obviously each component of G must intersect some set $L(i_1, i_2, \cdots, i_j)$ and must be contained in that set. Since there are more sets in L than there are components in G there exists a set $L^* \equiv L(I_1, I_2, \cdots, I_j)$ such that $L^*G=0$. However $L=L^*+(L-L^*)$ is a separation of L and no point-orbit in G intersects L^*. This is impossible because of Lemma 1. Thus Case (ii) never holds and Lemma 2 is true.

The next and final lemma will be recognized as one having the same conclusion as Lemma 1 established however from fewer assumptions.

LEMMA 1*. *If M is a compact metric space, $T(M)=M$ a pointwise periodic transformation, $\{G_i\}$ a convergent sequence of closed component-orbits under T with limit set L, and if G_x denotes any point-orbit in L, then L has the property that for every separation*

$$L = L^1 + L^2$$

the sets $G_x \cdot L^1$ and $G_x \cdot L^2$ are non-vacuous.

PROOF. Assume contrary to the lemma that $L=L^1+L^2$ is a separation of L and there exists a point x in L such that the point-orbit G_x is contained in one side, say L^1, of the separation. In these circumstances it is possible to choose a subsequence $\{G_{i_k}\}$ (of the given sequence of component-orbits) which in turn allows us to choose points $x_k \in G_{i_k}$ such that the *point*-orbits $\{G_{x_k}\}$ *converge* to a limit set, say L^*, and the points $\{x_k\}$ converge to the point x, which defines G_x. Since the point-orbits converge, we may use Lemma 2 which informs us that the limit set L^* is a component-orbit. Moreover each of the *finite number* of components of L^* may be taken as one side of a

separation of L^* and this means, in view of Lemma 1 (or Lemma 2) and the fact that x is in L^*, that each component Y of L^* is intersected by G_x. Now G_x is contained in L^1 hence Y is contained in L^1 and in turn L^* is contained in L^1.

On the other hand, using the fact that we have a separation of L and L is closed there exists a neighborhood V containing L^2 having the properties that the boundary $F(V)$ is disjoint with L and V is disjoint with L^1.

From this it follows that: (α) *the neighborhood V has at most a finite number of points in common with the compact closed set $\sum G_{x_k}+L^*$.* To show this, consider any infinite set in the product. This set determines a limit point z which belongs to L^* and hence to L^1. Such a point z is in $\overline{V}=V+F(V)$ and not in $F(V)$ (since z is in L), hence z is in V. This means z is in V and L^1 both which is impossible.

Let p be any point of L^2 and consider V as a neighborhood of p. It follows that there exists an integer K_1 such that $k>K_1$ implies $G_{i_k}\cdot V$ non-vacuous since $\{G_{i_k}\}$ converges to L. Hence there exists a component Y_k of G_{i_k} having the property that $Y_k\cdot V$ is non-vacuous for each $k>K_1$.

If we recall the introductory remarks concerning the relation between component-orbits and the point-orbits which they contain we see that the inequality $G_{x_k}\cdot\Gamma_k\neq 0$ holds for every component Γ_k in the orbit G_{i_k} since $x_k\in G_{i_k}$. Hence we have $G_{x_k}Y_k\neq 0$ for the particular (when k is fixed) component selected in the paragraph above. This latter statement concerning Y_k together with the fact (statement (α) in italics above) that $V\cdot(\sum G_{x_k}+L^*)$ is at most finite tell us there exists an integer K_2 such that for $k>K_2$ the component Y_k intersects the complement of V.

Combining the two properties associated with K_1 and K_2 it is clear that for $k>\max(K_1, K_2)$ each Y_k intersects V and the complement of V. Hence for each k there exists a point $y_k\in Y_k$ such that $y_k\in F(V)$. This leads immediately to the conclusion that there exists a point y common to L and $F(V)$. By the choice of V this cannot be true and the proof is complete. With the aid of Lemma 1* we now prove the new theorem which yields the same conclusion as Lemma 2.

THEOREM. *If M is a compact metric space, $T(M)=M$ a pointwise periodic transformation and $\{G_i\}$ a convergent sequence of closed component-orbits under T with limit set L, then L is likewise a closed component-orbit.*

PROOF. *Case* (i): *There exists a component X of L such that X is isolated in L.*

In this case $L=G+(L-G)$, where G is the component orbit of X under T, is either a separation of L or the left member is empty. This is true because G is both open and closed in L. In view of Lemma 1* the left side is empty; hence $L=G$ as stated by the theorem.

Case (ii): *No component of L is isolated in L.*

Since L is not connected let $L=L(1)+L(2)$ be some separation of L. By the reasoning applied in Case (i) there exists no isolated component in either $L(1)$ or $L(2)$. Now let $L(1)=L(1, 1)+L(1, 2)$ and $L(2)=L(2, 1)+L(2, 2)$ be separations of these sets and express L as the sum of the four sets with double indices. Proceeding this way the proof is the same as that in Case (ii) of Lemma 2.

REMARKS. Let $\{G_{x_k}\}$ be a sequence of point-orbits such that the number of points in G_{x_k} is bounded independent of k. It is a well known fact that if $\{G_{x_k}\}$ converges to a limit set L then L is also a point-orbit having the same bound. The theorem of this note and the simple result just mentioned are obviously of the same type in that each characterizes the limit set by the exact property it assumes for each element of a given sequence. This analogy is supported by the fact that if there is a common bound for the number of components in the orbits of the sequence the conclusion of the theorem can be augmented to include this bound for the component number of the limit set. On the other hand any improvement in the conclusion of Lemma 2 which might be suggested by this esthetic point of view (because the elements of the sequence have a minimal character) seems doomed to failure. These four possibilities form a complete set of the bounded, non-bounded, minimal and non-minimal types of sequence.

These remarks suggest that we examine certain theorems looking toward a possible analogy with *periodic* homeomorphisms—that is, the type of homeomorphism for which there exists a positive integer N such that every point-orbit contains at most N points. In this connection let us suppose that M is a compact metric space and K is any subset of M such that $\overline{K}=M$. If $T(K)=K$ is periodic on K and $S(M)=M$ is any homeomorphic extension of T, *then S is periodic on* M. This conclusion follows by an elementary demonstration. Homeomorphic extension is used here as meaning $S(M)=M$ is a homeomorphism such that $S(x)=T(x)$ for $x\in K$. The author feels that it is not only natural to ask about such extensions for the pointwise periodic type of homeomorphism, but feels called upon to apologize for the relatively minor position the question must occupy in a note such as this. The example given below showing that the most obvious analogue of the extension theorem above does not hold came up in a

discussion group and it was decided on an intuitive basis that the extension would be pointwise periodic. Thus one may see in this example something of the inspiration for the theory of this note, a concrete expression of that theory, and at the same time a refutation of the intuitive decision.

EXAMPLE. Let a_1b_1 denote an arc with end points a_1 and b_1. Similarly let $a_{11}b_{11}$ and $a_{12}b_{12}$ be arcs with end points a_{11}, b_{11} and a_{12}, b_{12} respectively. If we identify the 3 points $b_1=a_{11}=a_{12}$ and identify no other points, we obtain a triod; that is, a Y shaped figure in which three otherwise distinct arcs have a common end point. The end points of this triod are a_1, b_{11} and b_{12}. Choose four more arcs calling them $a_{11i}b_{11i}$ and $a_{12i}b_{12i}$ where $i=1, 2$. Identify the end points $b_{11}=a_{111}=a_{121}$ and also the end points $b_{12}=a_{121}=a_{122}$ making however no other identification. This process can be continued indefinitely, the general rule being to identify the a-end points of exactly two new arcs with a particular b-end point of the dendrite obtained in the previous stage. (Note that a_1 remains as an end point at all times.) The completed figure is to be a dendrite such that all end points other than a_1 lie on the Cantor discontinuum (middle third set) of some linear interval. The discontinuum is chosen in advance, the arcs added at each stage all have the same diameter, say δ_n, where δ_n approaches zero as n increases, and the arcs as added "reach" toward, but never intersect the discontinuum. This is a well known construction usually used to illustrate a dendrite with uncountably many end points.

We now denote the complete dendrite by D, the Cantor set of end points by C, and let $K=D-C$. A homeomorphism $T(K)=K$ is defined as follows: T leaves each point of the arc a_1b_1 fixed. The arc $a_{11}b_{11}$ is sent into the arc $a_{12}b_{12}$ so that $T(b_{11})=b_{12}$ and $T(b_{12})=b_{11}$ and in general $p(T, x)=2$ for $x\in a_{11}b_{11}-a_{11}$. Note that $a_{11}=a_{12}=b_1$ is fixed. We will call this the second stage. The four arcs $a_{1jk}b_{1jk}$ for $j=1, 2$ and $k=1, 2$ are permuted by T so that $p(T, x)=4$ for $x\in a_{1jk}b_{1jk}-a_{1jk}$ while $T(b_{11})=b_{12}=T(a_{111})=a_{121}=a_{122}$ has period 2 as was defined previously. This is the third stage. In general the arcs added at the nth stage suffer a permutation among themselves of the period 2^{n-1} and the transformation agrees with the part described for the $(n-1)$th stage in the natural way indicated by the common points.

It is clearly true that $T(K)=K$ so defined and the associated inverse $T^{-1}(K)=K$ homeomorphism are each uniformly continuous on K. If necessary D can be constructed in the unit square using straight line segments. The high degree of symmetry allows T to act as a "rigid motion" from which the uniform continuity is apparent. It follows that $T(K)=K$ may be extended to a homeomorphism $S(D)=D$

defined over all of D. However S is not pointwise periodic for

$$G_{b_1} = b_1, \qquad G_{b_{11}} = b_{11} + b_{12},$$

and so on, is a sequence of point-orbits which may be considered as component-orbits relative to themselves. This sequence converges to C and C is not the orbit of one of its components; that is, not the orbit of one of its points since it contains uncountably many points. The local connectivity, symmetry, and so on, of this example indicate that any conditions on $\overline{K} = M = D$ that are sufficient to make S inherit the pointwise periodic property must be highly restrictive.

THE UNIVERSITY OF VIRGINIA AND
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CORRECTION TO "CLOSURE OF PRODUCTS OF FUNCTIONS"[1]

D. G. BOURGIN

Page 811, line 7. *Replace* $F(t)$ *by* $c(t)F(t)$.

Page 811, line 10. *Replace* corresponding *by* largest.

Add to postulate (a) $T(E)$ admits no finite basis.

Page 813, lines 13 and 14. *Replace* (b') *and* (c') *by* (b'') *and* (c'').

In footnote 13 *replace* E *by* I_2 *and* The sets $\cdots$ also *by* The sequence $\{t^r s^p\}$ $\cdots$ also.

Page 814, line 25. *Replace* the set $F(t)$, $\phi_\sigma(t)$, $\sigma = 1, \cdots, n$, is linearly independent *by* $F(t)$ is not linearly dependent on $\phi_\sigma(t)$, $\sigma = 1, \cdots, n$.

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[1] This Bulletin, vol. 46 (1940), pp. 807–815.

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This prize was founded in memory of Professor Maxime Bôcher. It is awarded every five years for a notable research memoir in analysis which has appeared during the preceding five years in a recognized journal published in the United States or Canada; the recipient must be a member of the Society, and not more than fifty years old at the time of publication of his memoir.

First (Preliminary) Award, 1923: To G. D. Birkhoff, for his memoir *Dynamical systems with two degrees of freedom.*

Second Award, 1924: To E. T. Bell, for his memoir *Arithmetical paraphrases,* and to Solomon Lefschetz, for his memoir *On certain numerical invariants with applications to abelian varieties.*

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These prizes were founded in honor of Professor Frank Nelson Cole, on the occasion of his retirement as Secretary of the American Mathematical Society and editor of the Bulletin, after twenty-five years of service; the fund was later doubled by his son, Charles A. Cole. They are awarded at five-year intervals for contributions to algebra and the theory of numbers, respectively, under restrictions similar to those for the Bôcher prize.

First Award, 1928: To L. E. Dickson, for his book *Algebren und ihre Zahlentheorie,* Zurich, 1927.

Second Award, 1931: To H. S. Vandiver, for his several papers on Fermat's last theorem published in the Transactions of the American Mathematical Society and the Annals of Mathematics during the last five years, with special reference to a paper entitled *On Fermat's last theorem,* which appeared in volume 31 of the Transactions.

Third Award, 1939: To A. A. Albert, for his papers on the construction of Riemann matrices, published in volumes 35 and 36 of the Annals of Mathematics.

The Eliakim Hastings Moore Fund.

This fund was founded in 1922 in honor of Professor Eliakim Hastings Moore, on the occasion of the twenty-fifth anniversary meeting of the Chicago Section of the American Mathematical Society. The interest on the fund is to be used at the discretion of the Council of the Society, for the publication of important mathematical books or memoirs, or the award of prizes.

The Marion Reilly Fund.

Dean Marion Reilly willed to the American Mathematical Society a portion of her estate to be used for the advancement of research in pure mathematics. Thus far the Society has received approximately $16,800. The estate is not entirely settled, and the Society will receive additional funds within the next few years.

The Ernest William Brown Fund.

From the will of Professor E. W. Brown, a fund of $1000 is available, the interest on which, at the discretion of the Council, can be used for the furtherance of such mathematical interests as (a) the publication of important mathematical books, memoirs, and periodicals, and (b) lectures to be delivered on special occasions by invited guests of the Society.

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(Rates for other years may be obtained from the Treasurer.)

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25	$143.37	55	$94.36
30	137.93	60	82.74
35	131.48	65	70.71
40	123.88	70	58.81
45	115.13	75	47.56
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W. L. Graves

Warrensburg
DeVore
J. H. Scarborough
Urban

MONTANA

Bozeman
Hurst
H. M. Schwartz

Helena
Canning
Topel

Missoula
E. F. A. Carey
Chatland
Lennes
A. S. Merrill

NEBRASKA

Lincoln
Huff

Union College
Ogden

University of Nebraska
Basoco
Brenke

C. C. Camp
Candy
Collins
Doole
Dribin
Gaba
T. A. Pierce
Rock
Runge
Omaha
Earl
Fitzpatrick
Wayne
J. W. Boyce
Hove

NEVADA

Reno
Beesley
Palm
Ross
Vance
F. Wood

NEW HAMPSHIRE

Concord
G. M. Conwell
Durham
D. C. Lewis
Slobin
Solt
Hanover
F. M. Morgan
Dartmouth College
B. H. Brown
Forsyth
C. N. Haskins
Mathewson
F. W. Perkins
R. Robinson
Silverman
C. E. Wilder

NEW JERSEY

Barber
Pratt
Bellep'ain
Durell
Hackettstown
LaMotte
Hightstown
Harrison
Huntley
Litterick
Hoboken
Haight
Stevens Institute of Technology
H. N. Davis
Deimel
Hazeltine
Korn
Olmstead
Jersey City
McGrath
Polachek
Montclair
Bull
Mountain Lakes
S. E. Hotelling
Newark
Charton
Conkling
H. C. Levinson
Rhodes
J. S. Thompson
Widmark
Newark College of Engineering
Fithian
Prudential Insurance Company
Bartleson
Mosesson
New Brunswick
Brasefield
Bunyan
H. S. Grant
Maker
Meder
R. Morris
C. A. Nelson
M. S. Robertson
Starke
Walter
C. R. Wilson
Paterson
Helly
Princeton
Institute for Advanced Study
J. W. Alexander
Ambrose
Bargmann
Bergmann
A. T. Brauer
Fubini
Gödel
Halmos
M. H. Heins
Maharam
W. Mayer
M. Morse
von Neumann
A. Schwartz
S. Sherman
Thrall
Veblen
H. Weyl
Princeton University
E. P. Adams
Bochner
Bohnenblust
A. A. F. Brown
Christie
A. Church
W. J. Dixon
L. P. Eisenhart
Elveback
Flood
Gillespie
Lefschetz
B. McMillan
Muhly
H. P. Robertson
Shannon
Singleton
Strodt
Tompkins
A. W. Tucker
Tuckerman
Tukey
Wallace
Wallman
Wedderburn
Wigner
Wilks
J. D. Williams
South Orange
L. M. Rauch
Stanwick
Trenton
Segal
Shuster
Upper Montclair
G. A. Campbell
New Jersey State Teachers College
D. R. Davis
Fehr
E. H. C. Hildebrandt
Mallory
Westmont
M. G. Epstein
West Orange
Edison

NEW MEXICO

Albuquerque
Barnhart
Newsom
Las Vegas
Rodgers
Portales
MacKay
Roswell
Alden
State College
Swingle

NEW YORK

Albany
Frankel
Hallett
Johannes
King
W. R. Thompson
New York State College for Teachers
Beaver
Birchenough
DoBell
Lester
E. C. Stokes
Alfred
Lowenstein
Aurora
M. P. Hollcroft
Wells College
Carroll-Rusk
T. R. Hollcroft
D. Manning
Beacon
H.-K. Hammer
Brooklyn
E. M. Blake
Deutsch
Douglas
Gilbarg
Groat
Holly
Lassar
L. L. Locke
Supnick
Wynne
Brooklyn College
Boeker
Borofsky
Boyer
Feld
Fialkow
Fleisher
Gorn
H. M. Griffin
Harkin
R. A. Johnson
Kennison
J. P. Kormes
A. W. Landers
MacNeish
A. J. Maria
D. H. Maria
L. T. Moore
Prenowitz
M. Richardson
J. E. Rosenthal
J. Singer
Sohn
J. Wolfe
Woodbridge
Long Island University
H. G. Lieber

L. R. Lieber
Polytechnic Institute of Brooklyn
W. J. Berry
Bond
Fenn
Greeley
Whitford
Pratt Institute
Cowles
J. E. Thompson
St. John's University
Cowan
St. Joseph's College for Women
Larkin
Buffalo
Coombs
O'Brian
Podmele
D'Youville College
M. M. Maloney
University of Buffalo
Gehman
Montague
E. R. Ott
Pound
Canton
Bates
Cedarhurst
R. F. Koch
Clinton
H. S. Brown
W. M. Carruth
Ferry
Crown Point
Henderson
Elmira
Suffa
Far Rockaway
L. Moore
Flushing
Hofmann
Queens College
R. G. Archibald
A. B. Brown
S. S. Cairns
Cope
N. A. Hall
Hartman
Hoffmann
Raudenbush
Sard
M. M. Sullivan
M. H. Turner
Zippin
Garden City
J. Bowden
Riblet
Geneva
Durfee
Hubbs
S. T. Parker
Hamilton
Aude
Munshower
Wardwell
Hempstead
Eaton
Stevens
Ithaca
E. A. Miller
Cornell University
Agnew
Bissinger
Carver
J. H. Curtiss
Ficken
Flexner
Givens
Golomb
Goodier
W. J. Harrington
Herzog
Hurwitz
B. W. Jones
Kac
Karapetoff
Lawrence
Malti
D. S. Miller
Monroe
Randolph
Rosser
Sharpe
V. Snyder
Stephany
G. B. Thomas
Trevor
G. L. Walker
R. J. Walker
Wray
Keuka Park
H. F. Archibald
Long Island City
Berman
Killian
New York
Alsberg
J. W. Arnold
Arrow
Barlow
M. F. Becker
Blanch
Boehm
M. A. Bowden
Brodie
L. Burgess
R. W. Burgess
Byerly
Capecelatro
R. V. Carpenter
Cosby
Crane
Dix
Downey
G. A. Ellis
F. H. Fowler
Frey
R. L. Gilman
Greitzer
Ingram
Isaacs
Jacobus
Joffe
Kagno
S. Y. Keyser
B. F. Kimball
Kiss
M. Kormes
Laderman
H. Levi
D. P. Ling
L. A. Lorch
Lydenberg
Mirick
Mittleman
Morrison
Neuman
J. R. Newman
Parsons
E. A. Porter
Preston
R. B. Robbins
Roos
Roosevelt
Scherk
Serbin
Shamos
R. C. Shook
L. G. Simon
L. G. Simons
Snook
Spitzbart
Steinman
Sternberg
Tanzola
Tepping
Traber
Vitale
W. Weaver
A. W. Whitney
E. C. Williams
York
American Mathematical Society
E. M. Hull
O. L. Wheeler
American Telephone and Telegraph Company
Belcher
Carson
Halbert
Bell Telephone Laboratories
Bode
Burton
Dietzold
R. M. Foster
Fry
Gray
Hebbert
Hoyt
P. C. Jones
MacColl
Mead
Molina
Riordan
Sandalls
Schelkunoff
Shewhart
Stibitz
Sunde
Vaage
Wise
Zobel
College of Mt. St. Vincent
O'Brien
Sheridan
College of the City of New York
Barber
Garrison
Gill
C. C. Grove
Hubert
Linehan
P. Newman
Philip
Post
F. G. Reynolds
S. L. Robinson
Shiffman
Wetzel
Wirth
Wright
Columbia University
A. C. Berry
Clingan
Crucet
Fiske
Fite
Gourin
Hawkes
H. Hotelling
Ingersoll
Jablon
Kasner
C. J. Keyser
Kolchin
Koopman
E. R. Lorch
McKean
Mullins

F. J. Murray
Northcott
Pfeiffer
C. L. Poor
Reeve
Rehberg
Ritt
Ruger
Schatten
Serghiesco
Siceloff
D. E. Smith
P. A. Smith
Sutherland
Upton
Wald
H. B. Williams

Cooper Union
Lehmann
MacDonald
F. H. Miller
Reddick
Sugar

Equitable Life Assurance Society
Murphy
Steinhaus

Fordham University
Kirby
Kubis
Murtaugh
Nardelli

Hunter College
R. L. Anderson
Aroian
Bradley
Jewell H. Bushey
J. H. Bushey
Cooper
Darkow
Guggenbühl
F. C. Hall
L. S. Hill
Infeld
M. K. Landers
M. Levin
I. C. McLaughlin
MacDuffee
M. S. Rees
Tuller
Weisner
A. M. Whelan

Manhattanville College of the Sacred Heart
Dever
Mira

Metropolitan Life Insurance Company
Alfieri
Bassford
Fisanick
J. A. Hamilton
Hohaus
Lotka
Penney

Mutual Life Insurance Company
Carner
Monsky
Strong

New York University
Adler
F. Bernstein
Cooley
Courant
Davids
Farnum
Flanders
Friedrichs
Graham
F. W. John
M. Kline
McKinsey
Mintzer
Payne
A. S. Peters
R. G. Putnam
I. F. Ritter
H. E. Robbins
S. G. Roth
Schlauch
Stoker
Tilley
Wahlert
Yanosik

Webb Institute of Naval Architecture
B. C. Keeler

Yeshiva College
Ginsburg
Lowan

Niagara University
Banks

Oneonta
K. A. Bush
A. V. Newton

Perry
G. W. Walker

Poughkeepsie
L. D. Cummings
Hopper
G. Smith
M. E. Wells
H. S. White
Wishard

Richmond
Wernick
Wolfowitz

Riverdale-on-Hudson
H. B. Mitchell

Rochester
W. Betz
Chesna
Herzberger

University of Rochester
Atkins
Galbraith
Gale
J. W. Green
Long
Seidel
Watkeys

Rockville Centre
A. M. C. Grant

St. Bonaventure
Scheier

Schenectady

General Electric Company
Alger
Poritsky

Union College
Burkett
Farrell
A. H. Fox
Garis
D. S. Morse
A. D. Snyder

Scotia
Leibing

Syracuse
Metzler
Roe

Syracuse University
A. D. Campbell
I. S. Carroll
Cowgill
Decker
J. E. Hart
Harwood

Troy
E. B. Allen
D. B. Ames
J. D. Campbell
Huston
Lengyel
Rempfer
Street

Warsaw
Schneckenburger

West Point
H. Jones

Willseyville
Seely

NORTH CAROLINA

Asheville
Frank

Chapel Hill
Blau
Browne
E. A. Cameron
Garner
Hobbs
Hoyle
Hurewicz
Jacobson
Lasley
Linker
Mackie
M. H. Williams

Clinton
Culbreth

Durham

Duke University
Boas
Carlitz
Civin
Dressel
Elliott
George
Gergen
J. A. Greenwood
Hickson
Jarnagin
Layne
E. R. C. Miles
W. L. Morris
J. H. Roberts
J. M. Thomas
L. I. Wade

North Carolina College for Negroes
A. Elder

Fallston
Lee

Greensboro
Pegram
Thaxton

Greenville
ReBarker

Raleigh
Efird

North Carolina State College
Bullock
Cell
J. M. Clarkson
H. A. Fisher
Gelbart
J. Levine
Mumford
Nahikian
H. V. Park
Winton

Salisbury
Dearborn
Patten
Wilson
R. E. Smith

NORTH DAKOTA

Fargo
I. W. Smith

OHIO

Ada
Fulton
Alliance
Hildner
Ashland
Stopher
Athens
Denbow
Marquis
D. D. Miller
F. W. Reed
Starcher
Bluffton
Hirschler
Bowling Green
Ogg
Overman
Cincinnati
Our Lady of Cincinnati College
Reingold
University of Cincinnati
I. A. Barnett
Brand
Farnau
Hancock
Horvay
Justice
Kersten
Lubin
Merriman
H. L. Miller
C. N. Moore
E. S. Smith
Szász
W. C. Taylor
Yowell
Western and Southern Life Insurance Company
Folz
Learson
Ryan
William J. Williams
Xavier University
Stechschulte
Cleveland
Burwell
Kraft
Case School of Applied Science
O. E. Brown
Burington
Dobbie
Focke
McCuskey
M. Morris
Rinehart
C. F. Thomas
C. C. Torrance
Fenn College
H. W. Alexander
W. A. Patterson
Tolar
Western Reserve University
M. G. Boyce
Musselman
Sauté
W. G. Simon
Cleveland Heights
E. M. Torrance
Columbus
G. E. Albert
Bamforth
Bareis
Blumberg
Hendrickson
Kuhn
LaPaz
Radó
Rasor
Reade
W. S. Snyder
Toops
J. H. Weaver
Wylie
Youngs
Dayton
Midgley
Delaware
Rowland
Gambier
Bumer
MacNeille
Granville
Kato
Ladner
Wiley
Green Camp
C. E. Leach
Hiram
Clarke
Kent
Brooks
F. Harshbarger
Manchester
E. J. Olson
Stelson
Marietta
T. Bennett
Sandt
Mt. St. Joseph
Corona
Oberlin
W. D. Cairns
M. M. Johnson
Sinclair
R. W. Wagner
Yeaton
Oxford
Miami University
W. E. Anderson
Pollard
Western College
Tappan
Painesville
R. M. Peters
South Euclid
Garvin
Springfield
Tripp
Toledo
Brandeberry
Wilberforce
R. Coleman
Davidson
Wilmington
Spinks
Wooster
C. O. Williamson
Yanney
Yellow Springs
Astrachan
Tornheim

OKLAHOMA

Edmond
L. W. Johnson
Langston
Forrest
Norman
LaFon
University of Oklahoma
Brixey
Court
Hassler
McFarland
Meacham
Randels
Reaves
Springer
Stillwater
J. Barnett
Diamond
O. H. Hamilton
P. E. Lewis
C. E. Marshall
Oboukhoff
H. W. Smith
Tulsa
Hodell
Ricker

OREGON

Corvallis
W. E. Milne
Scheffé
C. B. Smith
Sobczyk
Eugene
DeCou
Ghent
Kossack
Moursund
T. S. Peterson
Grants Pass
Feinler
Portland
Reed College
Goheen
F. L. Griffin
L. L. Johnson
R. A. Rosenbaum
University of Portland
Vassallo
Salem
C. F. Luther
Olshen

PENNSYLVANIA

Allentown
Deck
T. L. Koehler
Reichard
Altoona
Umberger
Annville
A. H. Black
Bethlehem
Beale
Cutler
Fort
Lamson
Latshaw
E. Park
Pitcher
Raynor
C. A. Shook
Smail
M. F. Smiley
Transue
Vanderslice
Bryn Mawr
Geiringer
Lehr
McKee
Oxtoby
A. P. Wheeler
California
Bernstorf

Carlisle
F. Ayres
Landis
Chambersburg
Davison
R. F. Johnson
Weeks
Clarion
C. E. Carey
Collegeville
Clawson
Dallas
Garrahan
Easton
R. P. Bailey
W. M. Smith
East Pittsburgh
Slepian
Emporium
Huntoon
Gettysburg
Arms
Livingood
Glenside
J. O. Patterson
Greensburg
McNeil
Harrisburg
Whited
Haverford
Allendoerfer
E. E. Betz
Gummere
Oakley
A. H. Wilson
Hazleton
Herpel
Immaculata
Anastasia Maria
Jenkintown
J. C. Durand
Kutztown
Knedler
Lancaster
W. R. Murray
LaPlume
Trapani
Latrobe
Yahner
Lewisburg
Drum
Gold
W. I. Miller
Lincoln University
Haviland
Meadville
Joseph
Narberth
W. M. Robertson
New Wilmington
H. L. Black
Philadelphia
Beamish
A. Blake
R. D. Brown
W. B. Campbell
Constable
Fudge
Hayward
McDonough
J. Sherman
Philadelphia College of Pharmacy and Science
Maddaus
Temple University
Gleason
Kratz
Lawton
C. N. Stokes
University of Pennsylvania
Beal
Caris
J. A. Clarkson
Erdös
H. B. Evans
J. R. Kline
Knobelauch
Laden
Lufkin
H. H. Mitchell
W. B. Morgan
Rademacher
Safford
Shohat
Shugert
Witmer
Pittsburgh
Briant
F. W. Brown
Cowley
Carnegie Institute of Technology
Clippinger
Dines
A. D. Hestenes
Hicks
R. P. Johnson
Landau
Morehead
Moskovitz
Neelley
E. G. Olds
Riggs
Rosenbach
Saibel
T. L. Smith
E. A. Whitman
Pennsylvania College for Women
Calkins
University of Pittsburgh
Culver
Foraker
Knipp
Speer
J. S. Taylor
Pottsville
Kendig
Kleinschmidt
Scranton
M. B. Walton
Selinsgrove
A. J. Smith
State College
H. B. Owens
Pennsylvania State College
T. Cohen
Curry
A. H. Frink
O. Frink
Gordon
C. H. Graves
Hagen
E. Johnson
Krall
E. I. Moody
F. W. Owens
Rupp
I. M. Sheffer
Stecker
C. C. Wagner
Swarthmore
Swann
Swarthmore College
Brinkmann
Dresden
Marriott
Uniontown
Sealander
Wallingford
J. A. Miller
Washington
Caton
Washington and Jefferson College
Atchison
Dorwart
Shaub

PHILIPPINE ISLANDS

Baguio
Melchor
Manila
Favila
Janairo
Jaramillo
Netzorg
Mapua Institute of Technology
Mijares
University of the Philippines
Gokhale
Perez
Virata

PUERTO RICO

Mayaguez
Morales
Sánchez-Díaz
Rio Piedras
Bobonis
Horne
Kuschke

RHODE ISLAND

Kingston
Pease
Providence
Brown University
C. R. Adams
R. C. Archibald
A. A. Bennett
Carlen
Coon
Demers
Feller
Forsythe
Frame
R. E. Gilman
D. W. Hall
Herriot
Lindsay
H. P. Manning
Neugebauer
R. G. D. Richardson
Schmidt
C. H. Smiley
Tamarkin
Providence College
McKenney
Précourt
West Warwick
Brant

SOUTH CAROLINA

Charleston
College of Charleston
Bushkovitch
The Citadel
Dve
Hedge
Columbia
Allen University
Peguese
University of South Carolina
J. B. Coleman
J. B. Jackson
Rock Hill
Royall
R. W. Stokes

Spartanburg
Ader
C. S. Pettis

SOUTH DAKOTA

Brookings
MacDougal
Walder
Wente
Rapid City
Petrie
Vermillion
Beaty

TENNESSEE

Knoxville
Eaves
Seward
Memphis
J. F. Locke
MacQueen
Nashville
J. K. Peterson
George Peabody College
Wren
Vanderbilt University
Dyer-Bennet
Miser
Sarratt
Ward-Belmont School
Clement
Falvey
Sewanee
Bruton

TEXAS

Abilene
Burnam
Mullings
Amarillo
Whetstone
Austin
Vickery
University of Texas
Batchelder
Coburn
W. B. Coleman
H. V. Craig
Decherd
Dodd
Ettlinger
R. E. Greenwood
F. B. Jones
Lubben
R. L. Moore
G. H. Porter
M. B. Porter
Sorgenfrey
Vandiver
College Station
Brewer
Chaney
Gross
Hurt
W. Jennings
Klipple
H. A. Luther
Corpus Christi
Seale
Crockett
Bright
Dallas
P. V. Montgomery
Southern Methodist University
Palmquist
P. K. Rees
Starr
Denton
E. H. Hanson
El Paso
Leech
Fort Worth
Sherer
Houston
Rust
Slotnick
Rice Institute
Bray
A. C. Dean
Leighton
Lovett
Ulrich
H. A. Wilson
University of Houston
Aucoin
J. D. Hutchinson
Huntsville
Querry
Jacksonville
H. E. Clarkson
Kingsville
E. C. Kennedy
Lubbock
Gilbert
Ollmann
E. L. Thompson
Wakerling
Marlin
Falconer
Mercedes
W. F. Shaw
San Antonio
Incarnate Word College
Gough
Our Lady of the Lake College
M. L. Hill
St. Mary's University
F. A. Becker
San Antonio Junior College
Hurry
Tulloch
Waco
Hedberg
Walter J. Williams
Waxahachie
G. A. Newton

UTAH

Logan
Bird
Tingey
Salt Lake City
J. L. Gibson
J. J. Hayes
Horsfall
Pehrson
S. S. Smith
Stafford

VERMONT

Burlington
Bullard
Fraleigh
Swift
Winooski Park
Gleyzal

VIRGINIA

Blacksburg
Hatcher
J. E. Williams
Charlottesville
M. J. Cox
Hedlund
Linfield
McShane
R. C. Morrow
F. G. Myers
Oglesby
Sparrow
P. A. White
G. T. Whyburn
Dahlgren
Barker
L. T. E. Thompson
Ettrick
R. R. McDaniel
Fredericksburg
H. C. Carter
Hampton
H. A. Perkins
Simond
Hollins College
H. W. Allen
Hopewell
Fazel
Lexington
Virginia Military Institute
Byrne
Knox
Purdie
Washington and Lee University
Paxton
Lynchburg
Larew
Wiggin
Richmond
E. J. Willis
University of Richmond
Gaines
E. K. Ritter
C. H. Wheeler
Staunton
M. E. Taylor
Sweet Briar
N. Cole
Morenus
Williamsburg
Stetson

WASHINGTON

Lacey
Raymond
Pullman
L. G. Butler
Colpitts
Hacker
Knebelman
Seattle
J. P. Dean
Seattle Pacific College
Beegle
University of Washington
C. R. Ballantine
J. P. Ballantine
Beaumont
Benson
Birnbaum
A. F. Carpenter
Cramlet
Jerbert
McFarlan
Magnusson
Moritz
Mullemeister
Neikirk
R. S. Phillips
Taub
Winger

Zuckerman
Walla Walla
Bratton
Highberg
Woodinville
Anthony

WEST VIRGINIA

Bethany
Cramblet
Huntington
Hackney
Montgomery
W. F. Smith
Morgantown
Colwell
H. A. Davis
Eiesland
C. N. Reynolds
B. M. Turner
Philippi
B. D. Grant
Wheeling
Bagby

WISCONSIN

Beloit
Bisshopp
H. H. Conwell
Kenosha
A. E. May
Madison
Kelly
University of Wisconsin
F. E. Allen
J. H. Bell
Breit
Colvin
Cronvich
Crow
C. Eisenhart
W. S. Erickson
H. P. Evans
Hyers
Ingraham
S. B. Jackson
R. E. Johnson
Kiokemeister
Kleene
Langer
Leavitt
Lindquist
Lindstrum
March
W. L. G. Mitchell
Morkovin
Pell
Slichter
W. N. Smith
E. S. Sokolnikoff
I. S. Sokolnikoff
R. D. Specht
B. M. Stewart
Trump
Ulam
E. B. Van Vleck
Varino
Zilmer
Milwaukee
D. L. Bernstein
P. H. Evans
Marquette University
Pettit
Wilczewski
Milwaukee-Downer College
Beckwith
State Teachers College
Knight
R. E. Norris
University of Wisconsin
Bardell
Battig
Marden
Nordhaus
Parkinson
W. E. Roth
Vass
L. A. Wolf
Oshkosh
Beenken
I. Price
Platteville
Harrell
River Falls
Eide
Superior
C. W. Smith
West De Pere
DeCleene
Sromovsky
Wisconsin Rapids
M. B. McMillan

WYOMING

Lamont
Bellamy
Laramie
Rechard

CANADA

Alberta

Airdrie
Derry
Edmonton
J. W. Campbell
A. J. Cook
Sheldon

British Columbia

Vancouver
D. Buchanan
Gage
R. Hull
S. A. Jennings
Nowlan

Manitoba

Winnipeg
N. R. Wilson

New Brunswick

Sackville
W. H. McEwen

Nova Scotia

Halifax
Adshead
Walmsley
Wolfville
Jeffery
Macphail

Ontario

Hamilton
Findlay
Kingston
Halperin
Matheson
N. Miller
Watts
London
R. H. Cole
Kingston
Ottawa
G. V. Brady
Wyman
Toronto
Wolfenden
Ontario College of Education
A. A. Grant
University of Toronto
Beatty
R. Brauer
Burk
Coxeter
DeLury
Gilchrist
Krieger
Pounder
D. A. F. Robinson
G. de B. Robinson
Synge
Waddell
W. J. Webber
Victoria College
S. H. Gould

Quebec

Lennoxville
A. V. Richardson
Montreal
MacLean
Pall
A. N. Shaw
C. T. Sullivan
W. L. G. Williams
Quebec
Tremblay

Saskatchewan

Saskatoon
R. D. James
G. H. Ling
Murdoch

ARGENTINA

Buenos Aires
Baidaff
Barral-Souto
González-Dominguez
Rosario
B. Levi

AUSTRALIA

Adelaide
Wilton
Brisbane
Raybould
Hobart
J. C. Jaeger
Melbourne
Cherry
Perth
Weatherburn
Sydney
Carslaw
Thorne

BELGIUM

Ostend
Bouckaert

CANAL ZONE

Gatun
Eberman

CHINA

Peking
Hsü
Shanghai
Fan

COLOMBIA

Bogota
R. H. Cramer

CUBA

Havana
Corral-Aléman

DENMARK

Copenhagen
Royal Technical College
Jessen
University of Copenhagen
Bohr
Nörlund

ENGLAND

Birmingham
G. N. Watson
Bristol
Fraser
Cambridge
J. S. Turner
University of Cambridge
Collingwood
P. Hall
G. H. Hardy
Ingham
Larmor
H. W. Richmond
Smithies
Devon
Perryman
Hindhead Surrey
Western
Leeds
W. P. Milne
London
Brill
Dalal
University of London
Semple
H. Simpson
Taussky
Manchester
Mordell
Oxford
A. L. Dixon
Frecheville
J. H. C. Whitehead
Stocksfield-on-Tyne
Hardcastle
Suffolk
Curjel

FINLAND

Helsingfors
Ahlfors

FRANCE

Paris
Catholic Institute
Potron
University of Paris
Cartan
Fréchet
Rennes
Chevalley

GERMANY

Freiburg i. Br.
Bolza
Giessen
Geppert
Hannover
Prange
Leipzig
Ackermann-Teubner
Hopf

GREECE

Athens
Lanckton

HUNGARY

Budapest
Féjer
C. Jordan de Kerékjárto
Szeged
Riesz

INDIA

Aligarh
Shah
Annamalainagar
Rao
Bangalore
Iyengar
Dacca
Vijayaraghavan
Delhi
Seth
Madras
Ananda-Rau
Poona
Paranjpye

IRAQ

Baghdad
Merrick

IRELAND

Dublin
A. J. McConnell
Rowe

ITALY

Milan
Ricci
Pisa
Tonelli
Rome
Levi-Civita
Volterra
Turin
Tricomi

JAPAN

Osaka
Nakayama
Sapporo
Kawaguchi
Sendai
Kubota
Takasu
Tokyo
Kuniyeda
Yoshida

LATVIA

Riga
Leimanis

MEXICO

Mexico
Salazar y Arce

NETHERLANDS

Delft
van Dantzig

NEW ZEALAND

Auckland
Forder
Dunedin
Martyn

NORTHERN IRELAND

Belfast
Gillis
McCrea
Povey

PALESTINE

Jerusalem
Levitzki
Nablus
Tukan
Petah Tikvah
Preiser

PERU

Lima
Catholic University of Peru
de Losada y Puga
University of San Marcos
García
Rosenblatt

SCOTLAND

Dundee
Copson
Edinburgh
Whittaker

SOUTH AFRICA

Cape Town
de Wet

STRAITS SETTLEMENTS

Singapore
Oppenheim

SWEDEN

Stockholm
Oseen

THAILAND

Bangkok
Vadhana

TURKEY

Istanbul
W. S. Kimball

U.S.S.R.

Kieff
Kryloff

WALES

Bangor
Berwick
Cardiff
Livens
Swansea
A. R. Richardson

BY-LAWS OF THE AMERICAN MATHEMATICAL SOCIETY

ARTICLE I

Officers

Section 1. The officers of the Society shall be a President, three Vice Presidents, a Secretary, four Associate Secretaries, a Treasurer, a Librarian, and four Editorial Committees, one of four members for the *Bulletin*, one of three members for the *Transactions*, one of three members for the *Colloquium Publications*, and one consisting of three representatives of the Society on the Board of Editors of the *American Journal of Mathematics*.

Section 2. It shall be a duty of the President to deliver an address before the Society at the close of his term of office or within one year thereafter.

Section 3. The Librarian shall have charge of arrangements for the exchange of the Society's publications.

ARTICLE II

Board of Trustees

Section 1. There shall be a Board of Trustees consisting of five trustees elected by the Society, in accordance with Article IV.

Section 2. The function of the Board of Trustees shall be to receive and administer the funds of the Society, to have full legal control of its investments and properties, to make contracts and, in general, to conduct all business affairs of the Society.

Section 3. The Board of Trustees shall have the power to appoint a manager and such assistants and agents as may be necessary or convenient to facilitate the conduct of the affairs of the Society, and to fix the terms and conditions of their employment. The Board may delegate to the officers of the Society duties and powers normally inhering in their respective corporative offices, subject to supervision by the Board. The Board of Trustees may appoint committees to facilitate the conduct of the business of the Society and delegate to such committees such powers as may be necessary or convenient for the proper exercise of those powers. Agents appointed, or members of committees designated, by the Board of Trustees need not be members of the Board.

Nothing herein contained shall be construed to empower the Board of Trustees to divest itself of responsibility for, or legal control of, the investments, properties and contracts of the Society.

ARTICLE III

Council

Section 1. The Council shall consist of the officers specified in Article I, of ex-secretaries who have served as Secretary for ten years or more, of ex-presidents for a period of six years after the expiration of their respective presidential terms, and of fifteen additional elected members chosen in accordance with Article IV, Section 2.

Section 2. Its function shall be to formulate and administer the scientific policies of the Society and to act in an advisory capacity to the Board of Trustees.

Section 3. At meetings of the Council five members shall constitute a quorum for the transaction of business. Between meetings business may be transacted by a mail vote. This shall require at least fifteen favorable votes and also that the number of these votes be three-quarters of those transmitted to the Secretary before the time announced for the closing of the polls. In case three or more members of the Council request postponement, at time of voting, action on the matter at issue shall be postponed until the next meeting of the Council, unless at the discretion of the Secretary the question is made the subject of a second letter vote, in connection with which brief statements of reasons, pro and con, are circulated to members.

ARTICLE IV

Election and Term of Office of Officers and Trustees

Section 1. The term of office shall be two years for the trustees, the President, the Vice Presidents, the Secretary, the Associate Secretaries, and the Treasurer; three years each for the Librarian, the Editorial Committees of the *Bulletin*, the *Transactions*, and the *Colloquium Publications*, and the fifteen elected members of the Council, five of the latter retiring annually; five years for the representatives of the Society on the Board of Editors of the *American Journal of Mathematics*. In every case, however, the trustees and officers shall continue until their successors shall have been duly elected and qualified. The President, the Vice Presidents, and the fifteen elected members of the Council shall not be eligible for immediate reelection to their respective offices.

Section 2. Election of trustees, officers, and members of the Council shall be by ballot at the Annual Meeting. An official ballot shall be sent to each member by the Secretary at least one month prior to the Annual Meeting, and such ballots, if returned to the Secretary in envelopes bearing the names of the voters and received prior to the closing of the polls, shall be counted at the Annual Meeting. Each ballot shall contain a name proposed by the Council for each office to be filled, with blank spaces in which the voter may substitute other names. A majority of all votes cast, whether in person or by mail, shall be necessary for election. In case of failure to secure a majority for any office, the members present at the Annual Meeting shall choose by ballot between the two having the highest number of votes.

Section 3. If the President of the Society die or resign, before the expiration of his term of office, the Council may, with the approval of the trustees, designate one of the Vice Presidents to serve as Acting President until the next Annual Meeting, when a President shall be elected by the Society. Such vacancies as may occur at any time among the other officers may be filled by the Council with the approval of the trustees.

Section 4. If any trustee die or resign during his tenure of office, the vacancy thus created shall be filled for his unexpired term by the Board of Trustees.

Section 5. If any elected member of the Council die or resign more than one year before the expiration of his term, the vacancy for the unexpired term shall be filled by the Society at the next Annual Meeting.

ARTICLE V

Election of Members

Section 1. Election of members shall be by vote of the Council.

Section 2. There shall be four classes of members, ordinary, contributing, institutional contributing, and sustaining.

Section 3. A firm, corporation, institution, association or individual interested in the support of mathematics, may be elected to sustaining membership. A sustaining member shall receive both the *Bulletin* and the *Transactions* (see Article X), and shall have the privilege of nominating one or more persons for election by the Council to ordinary membership in the Society, the number to be determined by the Council. Such nominations shall not be acted upon until at least thirty days after their presentation to the Council (at a meeting or by mail).

Section 4. A firm, corporation, institution, or association interested in the support of mathematics may be elected to institutional contributing membership. A contributing member shall receive the *Bulletin* or have the privilege, for each twenty-five dollars contributed, of nominating one person to ordinary membership in the Society. Such nominations shall not be acted upon until at least thirty days after their presentation to the Council (at a meeting or by mail).

Section 5. Except in the case of nominees of sustaining members and of institutional contributing members, application for admission to ordinary membership shall be made by the applicant, on a blank provided by the Secretary, and shall be approved by two members of the Society. Such applications shall not be acted upon until at least thirty days after their presentation to the Council (at a meeting or by mail), except in the case of members of other societies entering under special action of the Council.

Section 6. An ordinary member may become a contributing member by paying the dues for such membership.

ARTICLE VI

Dues

Section 1. Persons elected to ordinary membership in the Society by the Council, under the provisions of Article V, Section 5, shall be admitted to membership upon the payment, within sixty days of the date of their election, of an initiation fee of five dollars.

Section 2. The annual dues of persons elected by the Council to ordinary membership under the provisions of Article V, Section 5, shall be eight dollars, with the following exceptions: (1) during the first three years of membership, the annual dues shall be six dollars; a person shall be considered to have completed his first year of membership on January 1 following his election; (2) the amount of dues may be altered by reciprocity agreements with other societies; (3) the Council may make special rulings in exceptional cases, with the approval of the Board of Trustees; four dollars and fifty cents of the dues of each member shall be for a year's subscription to the *Bulletin*. Each new member shall pay in proportion to the unexpired fraction of the year at the time of his election.

Section 3. The minimum dues for a contributing member shall be fifteen dollars per year. Members may, upon their own initiative, pay larger dues.

Section 4. The dues for an institutional contributing member for any year shall not be less than twenty-five dollars. Institutions may pay larger dues.

Section 5. The dues of a sustaining member for any year shall not be less than one hundred dollars. A sustaining member who contributes annually at least five hundred dollars shall be designated as a Patron of the Society.

Section 6. Persons elected to ordinary membership as nominees of sustaining members or institutional contributing members under the provisions of Article V, Sections 3-4, shall not be required to pay an initiation fee. They shall not be required to pay dues, so long as it is agreed that they are designated by the

sustaining member or institutional contributing member under these provisions. If a nominee of a sustaining member or an institutional contributing member later becomes a dues-paying member, he shall pay dues at the rate of six dollars a year for the remainder (if any) of his first three years of membership, and eight dollars a year thereafter.

Section 7. If the annual dues of any member remain unpaid beyond a reasonable time, the Board of Trustees shall remove his name from the list of members, after due notice.

Section 8. Any member not in arrears of dues may become a life member on the payment of a sum to be determined in accordance with actuarial principles.*

Section 9. Any member who has been retired from active service on account of age and has been a member of the Society for twenty years or more may, upon notification to the Secretary of such retirement, have his dues remitted, on the understanding that he will thereafter receive the programs of the meetings, but not the *Bulletin*.

ARTICLE VII

Meetings

Section 1. The Annual Meeting of the Society shall be held between the fifteenth of December and the fifteenth of January next following. Notice of the time and place of this meeting shall be mailed by the Secretary or an Associate Secretary to the last known post office address of each member of the Society. The times and places of the Annual and other meetings of the Society shall be designated by the Council.

Section 2. The President may call a meeting of the Council whenever the affairs of the Society make it desirable; he shall call a meeting of the Council on written request of five of its members.

Section 3. The Board of Trustees shall have its regular Annual Meeting on the first day of the Annual Meeting of the Society. Special meetings of the Board of Trustees may be called by the Chairman of the Board upon three days' notice of such meeting mailed to the last known post office address of each trustee. He shall call a meeting upon the receipt of a written request of two of the trustees. Meetings for the transaction of business may also be held by common consent of all the trustees.

ARTICLE VIII

Sections

Whenever it shall appear to the Council that a sufficient number of members of the Society are desirous of conducting in any locality periodic meetings for the reading and discussion of mathematical papers, the Council may authorize the formation of a Section to be composed at each sectional meeting of such members of the Society as may be present; and the Council shall have the right to withdraw such authorization.

ARTICLE IX

Programs

Section 1. Papers intended for presentation at any meeting or sectional meeting of the Society shall be passed upon in advance by a program committee ap-

* See table on page 6.

pointed by or under the authority of the Council; and only such papers shall be presented as shall have been approved by such committee. Papers in form unsuitable for publication, if accepted for presentation, shall be referred to on the program as preliminary communications or reports.

Section 2. No matter of general business shall be considered at any meeting of the Society except the Annual Meeting without the recommendation of the Council.

ARTICLE X

Publications

Section 1. The Society shall publish an official organ called the *Bulletin of the American Mathematical Society* which shall be sent to each member of the Society; four dollars and fifty cents of the dues paid by each member shall be for a year's subscription to this publication. It shall publish a series of volumes called *Colloquium Publications*, which shall embody in book form new mathematical developments. It shall publish a journal, the object of which shall be to make known important researches presented at meetings of the Society; this journal shall be called the *Transactions of the American Mathematical Society*. It shall also coöperate in the conduct of the *American Journal of Mathematics*.

Section 2. The editorial management of the *Bulletin*, *Transactions*, and *Colloquium Publications* and the participation of the Society in the editorial management of the *American Journal of Mathematics* shall be in charge of committees as specified in Article I.

ARTICLE XI

Amendments

These By-Laws may be amended or suspended at any meeting of the Society on recommendation of the Council and by a two-thirds vote of the members present, provided notice of such proposed action and of its general nature shall have been given in the call for such meeting.

REPORT OF THE TREASURER FOR THE PERIOD DECEMBER 1, 1937 TO NOVEMBER 30, 1938

BALANCE SHEET

November 30, 1938

Assets

CASH IN BANKS:			
Corn Exchange Bank Trust Company	$ 9,531.78		
Bank for Savings	2,782.29		
Union Dime Savings Bank	5,986.55	$ 18,300.62	
INVESTMENTS*		124,318.99	
Total Assets			$142,619.61

Reserves and Surplus

RESERVES:			
Endowment Fund	$70,481.07		
Bôcher Fund	2,182.46		
Cole Fund	3,323.82		
Moore Fund	4,319.16		
Reilly Fund	17,733.35		
Life Memberships	5,763.88		
Sinking Fund	9,980.77		
Investment Losses	665.44		
International Congress	3,884.63		
Bulletin Reprinting and Index Fund	498.15		
Transactions Reprinting and Index Fund	220.43		
Transactions Life Subscription	116.60		
Colloquium	10,813.91	$129,983.67	
SURPLUS AT NOVEMBER 30, 1938		12,635.94	
Total Reserves and Surplus			$142,619.61

Statement of General Receipts and General Disbursements

GENERAL RECEIPTS:		
Dues from Ordinary Memberships	$13,406.16	
Dues from Contributing Memberships	1,124.96	
Dues from Institutional Memberships	6,455.00	
Initiation Fees	629.86	
Interest Income from Endowment Fund and General Investments	3,762.48	
Miscellaneous	566.25	
Total General Receipts		$25,944.71

* Value of investments, exclusive of Brennan, Conlon, Sullivan and Davenport bonds and mortgages, based on market quotations on November 30, 1938, is $88,666.80.

GENERAL DISBURSEMENTS:		
Secretaries (clerical)	$ 4,894.84	
Treasurer	615.65	
Officers' Traveling	476.40	
Office Furniture and Fixtures	169.22	
Library	436.24	
Membership Committee	28.93	
Gibbs Lecture	111.48	
American Journal Subvention	2,500.00	
Emergency	61.60	
Total General Disbursements		9,294.36
EXCESS OF GENERAL RECEIPTS OVER GENERAL DISBURSEMENTS		$16,650.35

SURPLUS AT DECEMBER 1, 1937		$18,142.71
Additions:		
Adjustment—Amortization of Investments	$ 70.79	
Adjustment—Life Membership Reserve	225.38	
Adjustment—Transactions Subscription Reserve	1.19	
Excess of General Receipts over General Disbursements (see above)	16,650.35	16,947.71
Total		$35,090.42
Deductions:		
Appropriation to:		
Investment Account for Bonds in Default	$ 3,600.00	
Reserve for Investment Losses	665.44	
Sinking Fund	4,000.00	
Bulletin Account	9,189.52	
Transactions Account	3,249.52	
Transactions Reprinting Account	500.00	
Colloquium Account	1,250.00	
SURPLUS AT NOVEMBER 30, 1938		$12,635.94

BULLETIN

Receipts:		
Subscription Sales	$ 1,694.64	
Reprint Sales	73.80	$ 1,768.44
Appropriation from General Receipts		9,189.52
Total		$10,957.96
Disbursements:		
Printing (2 numbers for 1937, 10 for 1938)	$ 6,757.79	
List of Members	620.13	
Salaries, Annuity and Pension	3,093.49	
Advertising	20.16	
Editors' Traveling	5.00	
Miscellaneous—Office Furniture, Supplies, etc.	461.39	10,957.96

Bulletin Reprinting and Index

BALANCE AT DECEMBER 1, 1937		$ 108.55
Sales of Back Volumes		389.60
BALANCE AT NOVEMBER 30, 1938		$ 498.15

Transactions

Receipts:		
Subscription Sales	$ 4,425.30	
Back Volume Sales	684.30	
Reprint Sales	208.22	
Contributions	17.00	$ 5,334.82
Appropriation from General Receipts		3,249.52
Total		$ 8,584.34
Disbursements:		
Printing (1 number for 1937, 5 for 1938)	$ 6,499.92	
Salaries, Annuity and Pension	1,755.45	
Advertising	20.16	
Miscellaneous—Postage, Supplies, etc.	308.81	8,584.34

Transactions Reprinting and Index

BALANCE AT DECEMBER 1, 1937		$ 38.59
Appropriation from General Receipts		500.00
Total		$ 538.59
Reprinting		318.16
BALANCE AT NOVEMBER 30, 1938		$ 220.43

Colloquium

BALANCE AT DECEMBER 1, 1937		$ 9,806.32
Receipts:		
Sales of Colloquium Volumes	$ 1,976.47	
Sales of Colloquium Volume 16 (Bliss) for National Research Council	133.39	
Appropriation from General Receipts	1,250.00	3,359.86
Total		$13,166.18
Disbursements:		
Printing (Volume 22)	$ 970.58	
Salaries	969.49	
Miscellaneous—Shipping, Supplies, etc. (including cost of shipping Volume 16 for National Research Council)	220.18	
Circulars and Advertising	62.13	
Payment to National Research Council for Volume 16 (Bliss)	114.89	
Colloquium Lecture Expenses	15.00	2,352.27
BALANCE AT NOVEMBER 30, 1938		$10,813.91

International Congress

Contributions	$ 4,500.00
Disbursements	615.37
BALANCE AT NOVEMBER 30, 1938	$ 3,884.63

Semicentennial

BALANCE AT DECEMBER 1, 1937		$ 2,000.00
Receipts:		
Registration Fees, Ticket Sales, etc.	$ 3,307.28	
Publications	1,019.31	4,326.59
Appropriation from Sinking Fund		2,019.23
Total		$ 8,345.82
Disbursements:		
Meeting Expenses	$ 3,650.57	
Publications	4,295.25	
Enlarged List of Members	400.00	8,345.82

Special Funds

	Total	Principal	Income
ENDOWMENT FUND			
Balance at December 1, 1937	$70,456.07	$70,456.07	
Additions:			
Income and Gifts	2,444.39	25.00	$2,419.39
Adjustment for Amortization of Investments	45.52		45.52
Total	$72,945.98	$70,481.07	$2,464.91
Deduction:			
Interest Distributed to General Income	2,464.91		2,464.91
Balance at November 30, 1938	$70,481.07	$70,481.07	
BÔCHER FUND			
Balance at December 1, 1937	$ 2,107.49	$ 1,188.00	$ 919.49
Additions:			
Income	73.59		73.59
Adjustment for Amortization of Investments	1.38		1.38
Balance at November 30, 1938	$ 2,182.46	$ 1,188.00	$ 994.46
COLE FUND			
Balance at December 1, 1937	$ 3,211.35	$ 2,093.13	$1,118.22
Additions:			
Income	110.39		110.39
Adjustment for Amortization of Investments	2.08		2.08
Balance at November 30, 1938	$ 3,323.82	$ 2,093.13	$1,230.69

MOORE FUND			
Balance at December 1, 1937	$ 4,173.89	$ 2,100.62	$2,073.27
Additions:			
Income	142.59		142.59
Adjustment for Amortization of Investments	2.68		2.68
Balance at November 30, 1938	$ 4,319.16	$ 2,100.62	$2,218.54
REILLY FUND			
Balance at December 1, 1937	$ 6,729.84	$ 6,531.00	$ 198.84
Additions:			
Income	510.56		510.56
Adjustment for Amortization of Investments	9.61		9.61
Received from Estate of Marion Reilly	10,483.34	10,483.34	
Balance at November 30, 1938	$17,733.35	$17,014.34	$ 719.01

BENNINGTON P. GILL
Treasurer

New York, N.Y.
November 30, 1938

REPORT OF THE AUDITING COMMITTEE
1938

We, the undersigned Auditing Committee, appointed by the American Mathematical Society, have this day audited the accounts of the Treasurer and in our opinion the following statement is correct:

CASH BALANCE AT DECEMBER 1, 1937:		
Corn Exchange Bank Trust Company	$3,579.88	
Bank for Savings	2,727.35	
Union Dime Savings Bank	5,868.30	$12,175.53
Cash Receipts—December 1, 1937 to November 30, 1938		46,612.49
Interest on Savings Accounts in Bank for Savings and Union Dime Savings Bank		173.19
Total		$58,961.21
Cash Disbursements—December 1, 1937 to November 30, 1938		40,660.59
CASH BALANCE AT NOVEMBER 30, 1938:		
Corn Exchange Bank Trust Company	$9,531.78	
Bank for Savings	2,782.29	
Union Dime Savings Bank	5,986.55	
		$18,300.62

RALPH G. ARCHIBALD
ALBERT E. MEDER, JR.

New York, N.Y.
December 20, 1938

REPORT OF THE TREASURER FOR THE PERIOD DECEMBER 1, 1938 TO NOVEMBER 30, 1939

BALANCE SHEET

November 30, 1939

Assets

CASH IN BANKS:			
Corn Exchange Bank Trust Company	$ 3,037.95		
City Bank Farmers Trust Company	5,554.34		
Bank for Savings	2,838.35		
Emigrant Industrial Savings Bank	7,508.88		
Providence Institution for Savings	3,500.00		
Union Dime Savings Bank	6,107.17	$ 28,546.69	
INVESTMENTS*		174,519.86	
Total Assets			$203,066.55

Reserves and Surplus

RESERVES:			
Endowment Fund	$70,506.07		
Bôcher Fund	2,171.62		
Brown Fund	1,012.52		
Cole Fund	3,466.50		
Moore Fund	4,504.31		
Reilly Fund	18,494.11		
Life Memberships	5,720.15		
Sinking Fund	10,258.72		
Investment Losses	1,655.49		
International Congress	5,716.62		
Bulletin Reprinting and Index Fund	311.76		
Transactions Reprinting and Index Fund	220.43		
Transactions Life Subscription	115.37		
Colloquium	8,075.38		
Mathematical Reviews	61,431.89	$193,660.94	
SURPLUS AT NOVEMBER 30, 1939		9,405.61	
Total Reserves and Surplus			$203,066.55

Statement of General Receipts and General Disbursements

GENERAL RECEIPTS:		
Dues from Ordinary Memberships	$13,541.71	
Dues from Contributing Memberships	1,163.92	
Dues from Institutional Memberships	6,530.00	
Initiation Fees	299.64	
Interest Income from Endowment Fund and General Investments	4,481.26	
Miscellaneous	456.24	
Total General Receipts		$26,472.77

* Value of investments, exclusive of Brennan, Sullivan and Conlon bonds and mortgages, based on market quotations on November 30, 1939, is $146,375.13.

GENERAL DISBURSEMENTS:		
Secretaries (clerical)	$ 5,500.00	
Treasurer	644.18	
Officers' Traveling	700.00	
Library	1,146.13	
Membership Committee	200.00	
American Journal Subvention	2,500.00	
Emergency	333.36	
Accrued Interest Purchased	191.45	
Total General Disbursements		11,215.12
EXCESS OF GENERAL RECEIPTS OVER GENERAL DISBURSEMENTS		$15,257.65

SURPLUS AT DECEMBER 1, 1938		$12,635.94
Additions:		
Adjustment—Amortization of Investments	$ 47.65	
Adjustment—Life Membership Reserve	273.74	
Adjustment—Transactions Subscription Reserve	1.23	
Excess of General Receipts over General Disbursements (see above)	15,257.65	15,580.27
Total		$28,216.21
Deductions:		
Appropriation to:		
Investment Account for Bonds in Default	$ 3,600.00	
Reserve for Investment Losses	990.05	
Bulletin Account	9,632.72	
Transactions Account	3,337.83	
Colloquium Account	750.00	
Mathematical Reviews Account	500.00	18,810.60
SURPLUS AT NOVEMBER 30, 1939		$ 9,405.61

BULLETIN

Receipts:		
Subscription Sales	$ 1,785.09	
Reprint Sales	81.39	$ 1,866.48
Appropriation from General Receipts		9,632.72
Total		$11,499.20
Disbursements:		
Printing (2 numbers for 1938, 10 for 1939)	$ 7,419.04	
Salaries, Annuity and Pension	3,223.84	
Advertising	37.82	
Editors' Traveling	210.50	
Miscellaneous—Office Furniture, Supplies, etc.	608.00	11,499.20

Bulletin Reprinting and Index

BALANCE AT DECEMBER 1, 1938		$ 498.15
Sales of Back Volumes		409.12
Total		$ 907.27
Reprinting		595.51
BALANCE AT NOVEMBER 30, 1939		$ 311.76

Transactions

Receipts:		
Subscription Sales	$ 4,846.73	
Back Volume Sales	714.28	
Reprint Sales	186.64	
Contributions	17.00	$ 5,764.65
Appropriation from General Receipts		3,337.83
Total		$ 9,102.48
Disbursements:		
Printing (1 number for 1938, 5 for 1939)	$ 6,792.53	
Salaries, Annuity and Pension	1,710.00	
Advertising	37.82	
Editors' Traveling	260.80	
Miscellaneous—Postage, Supplies, etc.	301.33	9,102.48

Transactions Reprinting and Index

BALANCE AT DECEMBER 1, 1938		$ 220.43
BALANCE AT NOVEMBER 30, 1939		$ 220.43

Colloquium

BALANCE AT DECEMBER 1, 1938		$10,813.91
Receipts:		
Sales of Colloquium Volumes	$ 2,404.51	
Sales of Colloquium Volume 16 (Bliss) for National Research Council	105.22	2,509.73
Appropriation from General Receipts		750.00
Total		$14,073.64
Disbursements:		
Printing (Volumes 23 and 24)	$ 4,559.28	
Salaries	1,006.25	
Miscellaneous—Shipping, Supplies, etc. (including cost of shipping Volume 16 for National Research Council)	240.00	
Circulars and Advertising	61.52	
Payment to National Research Council for Volume 16 (Bliss)	81.21	
Honorarium	50.00	5,998.26
BALANCE AT NOVEMBER 30, 1939		$ 8,075.38

MATHEMATICAL REVIEWS

Receipts:		
Contributions	$65,000.00	
Subscription Sales	328.53	
Income	379.85	$65,708.38
Appropriation from General Receipts		500.00
Total		$66,208.38
Disbursements:		
Salaries	$ 2,421.00	
Periodicals, Books, etc.	175.38	
Circulars and Advertising	411.42	
Traveling	362.86	
Office Furniture	824.23	
Miscellaneous	581.60	4,776.49
BALANCE AT NOVEMBER 30, 1939		$61,431.89

INTERNATIONAL CONGRESS

BALANCE AT DECEMBER 1, 1938		$ 3,884.63
Receipts:		
Contributions and Fees		3,805.00
Total		$ 7,689.63
Disbursements:		
Clerical	$ 1,251.23	
Stationery and Printing	436.93	
Traveling	61.15	
Miscellaneous—Supplies, Telegrams, Furniture, etc.	223.70	1,973.01
BALANCE AT NOVEMBER 30, 1939		$ 5,716.62

SPECIAL FUNDS

	Total	Principal	Income
ENDOWMENT FUND			
Balance at December 1, 1938	$70,481.07	$70,481.07	
Additions:			
Income and Gifts	3,017.78	25.00	$2,992.78
Adjustment for Amortization of Investments	31.82		31.82
Total	$73,530.67	$70,506.07	$3,024.60
Deduction:			
Interest Distributed to General Income	3,024.60		3,024.60
Balance at November 30, 1939	$70,506.07	$70,506.07	

BÔCHER FUND

Balance at December 1, 1938	$ 2,182.46	$ 1,188.00	$ 994.46
Additions:			
Income	88.22		88.22
Adjustment for Amortization of Investments	.94		.94
Total	$ 2,271.62	$ 1,188.00	$1,083.62
Deduction:			
Prize	100.00		100.00
Balance at November 30, 1939	$ 2,171.62	$ 1,188.00	$ 983.62

BROWN FUND

Received from Estate of E. W. Brown	$ 1,000.00	$ 1,000.00	
Income	12.37		$ 12.37
Adjustment for Amortization of Investments	.15		.15
Balance at November 30, 1939	$ 1,012.52	$ 1,000.00	$ 12.52

COLE FUND

Balance at December 1, 1938	$ 3,323.82	$ 2,093.13	$1,230.69
Additions:			
Income	141.18		141.18
Adjustment for Amortization of Investments	1.50		1.50
Balance at November 30, 1939	$ 3,466.50	$ 2,093.13	$1,373.37

MOORE FUND

Balance at December 1, 1938	$ 4,319.16	$ 2,100.62	$2,218.54
Additions:			
Income	183.20		183.20
Adjustment for Amortization of Investments	1.95		1.95
Balance at November 30, 1939	$ 4,504.31	$ 2,100.62	$2,403.69

REILLY FUND

Balance at December 1, 1938	$17,733.35	$17,014.34	$ 719.01
Additions:			
Income	752.76		752.76
Adjustment for Amortization of Investments	8.00		8.00
Balance at November 30, 1939	$18,494.11	$17,014.34	$1,479.77

BENNINGTON P. GILL
Treasurer

New York, N.Y.
November 30, 1939

REPORT OF THE AUDITING COMMITTEE
1939

We, the undersigned Auditing Committee, appointed by the American Mathematical Society, have this day audited the accounts of the Treasurer and in our opinion the following statement is correct:

CASH BALANCE AT DECEMBER 1, 1938:		
Corn Exchange Bank Trust Company	$9,531.78	
Bank for Savings	2,782.29	
Union Dime Savings Bank	5,986.55	$ 18,300.62
Cash Receipts—December 1, 1938 to November 30, 1939		116,562.39
Interest on Savings Accounts in Union Dime Savings Bank, Bank for Savings and Emigrant Industrial Savings Bank		185.56
Total		$135,048.57
Cash Disbursements—December 1, 1938 to November 30, 1939		106,501.88
CASH BALANCE AT NOVEMBER 30, 1939:		
Corn Exchange Bank Trust Company	$3,037.95	
Bank for Savings	2,838.35	
City Bank Farmers Trust Company	5,554.34	
Emigrant Industrial Savings Bank	7,508.88	
Providence Institution for Savings	3,500.00	
Union Dime Savings Bank	6,107.17	
		$ 28,546.69

RALPH G. ARCHIBALD
ALBERT E. MEDER, JR.

New York, N.Y.
December 18, 1939

PUBLICATIONS

JOURNALS

Bulletin of the American Mathematical Society.
Transactions of the American Mathematical Society.
Mathematical Reviews.

BOOKS

F. Klein, **The Evanston Colloquium Lectures on Mathematics**, 1894, x, 109 pp.; reprinted, 1911, xii, 109 pp. $1.00

Mathematical Papers Read at the International Mathematical Congress, 1896, xvi, 411 pp. 4.70

Semicentennial Publications.

1. R. C. Archibald, **A Semicentennial History of the American Mathematical Society, 1888-1938, with Biographies and Bibliographies of the Past Presidents**, 1938, xii, 261 pp., 30 plates. 3.00
2. **Semicentennial Addresses**, 1938, vi, 315 pp. 4.50

COLLOQUIUM SERIES

1. H. S. White, **Linear Systems of Curves on Algebraic Surfaces**; F. S. Woods, **Forms of Non-Euclidean Space**; E. B. Van Vleck, **Selected Topics in the Theory of Divergent Series and of Continued Fractions**; 1905, xii, 187 pp. 3.00
2.* E. H. Moore, **Introduction to a Form of General Analysis**; M. Mason, **Selected Topics in the Theory of Boundary Value Problems of Differential Equations**; E. J. Wilczynski, **Projective Differential Geometry**; 1910, x, 222 pp.
3$_1$. G. A. Bliss, **Fundamental Existence Theorems**, 1913, x, 107 pp.; reprinted, 1934, iv, 107 pp. 2.00
3$_2$. E. Kasner, **Differential-Geometric Aspects of Dynamics**, 1913, iv, 117 pp.; reprinted, 1934, iv, 117 pp. 2.00
4.* L. E. Dickson, **On Invariants and the Theory of Numbers**; W. F. Osgood, **Topics in the Theory of Functions of Several Complex Variables**; 1914, xviii, 230 pp.
5$_1$.* G. C. Evans, **Functionals and their Applications. Selected Topics, Including Integral Equations**, 1918, xii, 136 pp.
5$_2$.* O. Veblen, **Analysis Situs**, 1922, viii, 150 pp.; second ed., 1931, x, 194 pp.
6. G. C. Evans, **The Logarithmic Potential. Discontinuous Dirichlet and Neumann Problems**, 1927, viii, 150 pp. 2.00
7. E. T. Bell, **Algebraic Arithmetic**, 1927, iv, 180 pp. 2.70
8. L. P. Eisenhart, **Non-Riemannian Geometry**, 1927, viii, 184 pp.; reprinted, 1934, viii, 184 pp. 2.70
9. G. D. Birkhoff, **Dynamical Systems**, 1927, viii, 295 pp. 3.35
10. A. B. Coble, **Algebraic Geometry and Theta Functions**, 1929, viii, 282 pp. 3.35
11.* D. Jackson, **The Theory of Approximation**, 1930, viii, 178 pp.
12.* S. Lefschetz, **Topology**, 1930, x, 410 pp.

* Out of print.

13. R. L. Moore, **Foundations of Point Set Theory,** 1932, viii, 486 pp. 6.00
14. J. F. Ritt, **Differential Equations from the Algebraic Standpoint,** 1932, x, 172 pp. 2.70
15.* M. H. Stone, **Linear Transformations in Hilbert Space and their Applications to Analysis,** 1932, viii, 622 pp.
16. G. A. Bliss, **Algebraic Functions,** 1933, x, 218 pp. 3.35
17. J. H. M. Wedderburn, **Lectures on Matrices,** 1934, viii, 200 pp. 3.35
18. M. Morse, **The Calculus of Variations in the Large,** 1934, x, 368 pp. 5.35
19. R. E. A. C. Paley and N. Wiener, **Fourier Transforms in the Complex Domain,** 1934, viii, 184 pp. + portrait plate. 3.35
20. J. L. Walsh, **Interpolation and Approximation by Rational Functions in the Complex Domain,** 1935, x, 382 pp. 6.00
21. J. M. Thomas, **Differential Systems,** 1937, x, 118 pp. 2.00
22. C. N. Moore, **Summable Series and Convergence Factors,** 1938, vi, 105 pp. 2.00
23. G. Szegö, **Orthogonal Polynomials,** 1939, x, 401 pp. 6.00
24. A. A. Albert, **Structure of Algebras,** 1939, xii, 210 pp. 4.00
25. G. Birkhoff, **Lattice Theory,** 1940, vi, 155 pp. 2.50

* Out of print.